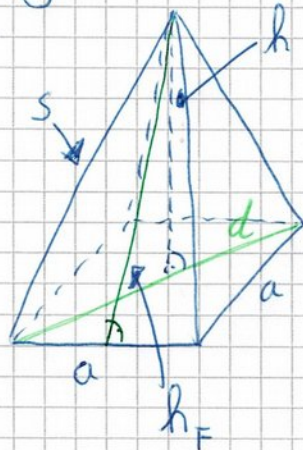


# AUFGABEN

1) Gegeben ist eine gerade Pyramide mit quadratischer Grundfläche. Berechne die fehlenden Angaben:



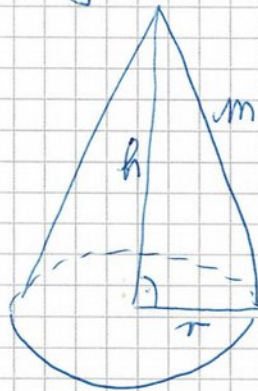
a)  $a = 4 \text{ cm}$        $G = ?$   
 $h = 10 \text{ cm}$        $M = ?$   
 $h_F = ?$        $O = ?$   
 $s = ?$        $V = ?$   
 $d = ?$

b)  $a = ?$        $G = 100 \text{ cm}^2$   
 $h = ?$        $M = ?$   
 $h_F = 8 \text{ cm}$        $O = ?$   
 $s = ?$        $V = ?$   
 $d = ?$

c)  $a = ?$        $G = ?$   
 $h = 12 \text{ cm}$        $M = ?$   
 $h_F = ?$        $O = ?$   
 $d = 8 \text{ cm}$        $V = ?$   
 $s = ?$

d)  $a = 6 \text{ cm}$   
 $h = ?$   
 $h_F = ?$        $s = ?$   
 $d = ?$   
 $G = ?$   
 $M = ?$   
 $O = ?$   
 $V = 1000 \text{ cm}^3$

2) Gegeben ist ein gerader Kegel. Berechne die fehlenden Angaben:



a)

$r = 6 \text{ cm}$	$G = ?$
$h = 10 \text{ cm}$	$M = ?$
$m = ?$	$O = ?$
	$V = ?$

b)

$r = ?$	$G = 100 \text{ cm}^2$
$h = ?$	$M = ?$
$m = 10 \text{ cm}$	$O = ?$
	$V = ?$

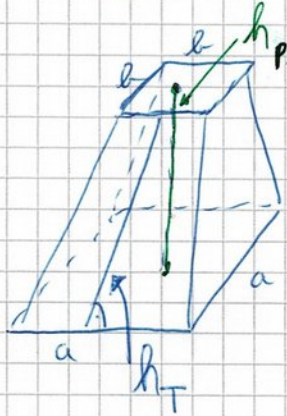
c)

$r = ?$	$G = ?$
$h = 12 \text{ cm}$	$M = ?$
$m = 16 \text{ cm}$	$O = ?$
	$V = ?$

d)

$r = 4 \text{ cm}$	$G = ?$
$h = ?$	$M = ?$
$m = ?$	$O = ?$
	$V = 1000 \text{ cm}^3$

3) Gegeben ist ein gerader Pyramidenstumpf mit quadratischer Grundfläche. Berechne die fehlenden Angaben:



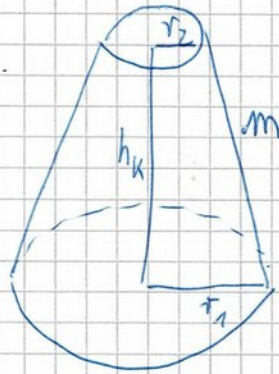
a)  $a = 8 \text{ cm}$   
 $b = 4 \text{ cm}$   
 $h_P = ?$   
 $h_T = 10 \text{ cm}$

$G = ?$   
 $S = ?$   
 $M = ?$   
 $O = ?$   
 $V = ?$

b)  $a = 5 \text{ cm}$   
 $b = ?$   
 $h_P = 10 \text{ cm}$   
 $h_T = ?$

$G = ?$   
 $S = 16 \text{ cm}^2$   
 $M = ?$   
 $O = ?$   
 $V = ?$

4) Gegeben ist ein gerader Kegelstumpf.  
Berechne die fehlenden Angaben:



a)

$r_1 = 10 \text{ cm}$	$G = ?$
$h_K = 16 \text{ cm}$	$S = ?$
$r_2 = 6 \text{ cm}$	$M = ?$
$m = ?$	$O = ?$
	$V = ?$

b)

$r_1 = 12 \text{ cm}$	$G = ?$
$h_K = ?$	$S = ?$
$r_2 = 2 \text{ cm}$	$M = ?$
$m = 16 \text{ cm}$	$O = ?$
	$V = ?$

# LÖSUNGEN

$$1a) \quad h^2 + \frac{1}{4}a^2 = h_F^2$$

$$10^2 + \frac{1}{4} \cdot 16 = h_F^2$$

$$100 + 4 = h_F^2$$

$$104 = h_F^2$$

$$10,2 \text{ cm} \approx h_F$$

$$h_F^2 + \frac{1}{4}a^2 = s^2$$

$$104 + \frac{1}{4} \cdot 16 = s^2$$

$$104 + 4 = s^2$$

$$108 = s^2$$

$$10,39 \text{ cm} \approx s$$

$$G = 4^2 = 16 \text{ cm}^2$$

$$M = 4 \cdot \frac{1}{2} \cdot a \cdot h_F$$

$$= 4 \cdot \frac{1}{2} \cdot 4 \cdot 10,2$$

$$= 81,6 \text{ cm}^2$$

$$O = G + M = 97,6 \text{ cm}^2$$

$$V = \frac{1}{3} \cdot a^2 \cdot h$$

$$= \frac{1}{3} \cdot 4^2 \cdot 10$$

$$= 53,3 \text{ cm}^3$$

$$2a^2 = d^2$$

$$2 \cdot 4^2 = d^2$$

$$2 \cdot 16 = d^2$$

$$32 = d^2$$

$$5,66 \text{ cm} \approx d$$

$$b) \quad G = 100 \text{ cm}^2$$

$$a^2 = 100$$

$$a = 10 \text{ cm}$$

$$h_F^2 = h^2 + \frac{1}{4}a^2$$

$$8^2 = h^2 + \frac{1}{4} \cdot 100$$

$$64 = h^2 + 25$$

$$39 = h^2$$

$$6,24 \text{ cm} = h$$

$$h_F^2 + \frac{1}{4}a^2 = s^2$$

$$8^2 + \frac{1}{4} \cdot 100 = s^2$$

$$64 + 25 = s^2$$

$$89 = s^2$$

$$9,43 \text{ cm} = s$$

$$2a^2 = d^2$$

$$2 \cdot 100 = d^2$$

$$200 = d^2$$

$$14,14 \text{ cm} = d$$

$$\begin{aligned}
 M &= 4 \cdot \frac{1}{2} \cdot a \cdot h_F \\
 &= 4 \cdot \frac{1}{2} \cdot 10 \cdot 8 \\
 &= 160 \text{ cm}^2
 \end{aligned}$$

$$O = G + M = 260 \text{ cm}^2$$

$$V = \frac{1}{3} \cdot a^2 \cdot h = \frac{1}{3} \cdot 100 \cdot 6,24 = 208 \text{ cm}^3$$

$$\begin{aligned}
 \text{c) } 2a^2 &= d^2 \\
 2a^2 &= 8^2 \\
 2a^2 &= 64 \\
 a^2 &= 32 \\
 a &= 5,66 \text{ cm}
 \end{aligned}$$

$$G = a^2 = 32 \text{ cm}^2$$

$$\begin{aligned}
 M &= 4 \cdot \frac{1}{2} \cdot a \cdot h_F \\
 &= 4 \cdot \frac{1}{2} \cdot 5,66 \cdot 12,33 \\
 &= 139,58 \text{ cm}^2
 \end{aligned}$$

$$O = G + M = 171,58 \text{ cm}^2$$

$$\begin{aligned}
 h_F^2 &= \frac{1}{4} a^2 + h^2 \\
 h_F^2 &= \frac{1}{4} \cdot 32 + 12^2 \\
 h_F^2 &= 8 + 144 \\
 h_F^2 &= 152 \\
 h_F &= 12,33 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 V &= \frac{1}{3} \cdot a^2 \cdot h \\
 &= \frac{1}{3} \cdot 32 \cdot 12 \\
 &= 128 \text{ cm}^3
 \end{aligned}$$

$$\begin{aligned}
 s^2 &= h_F^2 + \frac{1}{4} \cdot a^2 \\
 s^2 &= 152 + \frac{1}{4} \cdot 32 \\
 s^2 &= 152 + 8 \\
 s^2 &= 160 \\
 s &= 12,65 \text{ cm}
 \end{aligned}$$

$$d) V = \frac{1}{3} \cdot a^2 \cdot h$$

$$1000 = \frac{1}{3} \cdot 6^2 \cdot h$$

$$1000 = \frac{1}{3} \cdot 36 \cdot h$$

$$1000 = 12 \cdot h$$

$$83,3 \text{ cm} = h$$

$$h^2 + \frac{1}{4} \cdot a^2 = h_F^2$$

$$6943,89 + \frac{1}{4} \cdot 36 = h_F^2$$

$$6943,89 + 9 = h_F^2$$

$$6952,89 = h_F^2$$

$$83,38 \text{ cm} = h_F$$

$$s^2 = \frac{1}{4} a^2 + h_F^2$$

$$s^2 = \frac{1}{4} \cdot 36 + 6952,89$$

$$s^2 = 9 + 6952,89$$

$$s^2 = 6961,89$$

$$s = 83,44 \text{ cm}$$

$$2a^2 = d^2$$

$$2 \cdot 6^2 = d^2$$

$$2 \cdot 36 = d^2$$

$$72 = d^2$$

$$8,49 \text{ cm} = d$$

$$G = a^2 = 36 \text{ cm}^2$$

$$M = 4 \cdot \frac{1}{2} \cdot a \cdot h_F$$

$$= 4 \cdot \frac{1}{2} \cdot 6 \cdot 83,38$$

$$= 1000,56 \text{ cm}^2$$

$$O = G + M = 1036,56 \text{ cm}^2$$

$$2a) \quad r^2 + h^2 = m^2$$

$$6^2 + 10^2 = m^2$$

$$36 + 100 = m^2$$

$$136 = m^2$$

$$11,66 \text{ cm} = m$$

$$G = \pi r^2 = \pi \cdot 6^2 = 36\pi \approx 113,1 \text{ cm}^2$$

$$M = \pi r m = \pi \cdot 6 \cdot 11,66 = 69,96\pi \approx 219,79 \text{ cm}^2$$

$$O = G + M = 332,89 \text{ cm}^2$$

$$V = \frac{1}{3} \cdot \pi r^2 \cdot h = \frac{1}{3} \cdot \pi \cdot 6^2 \cdot 10 = 120\pi \approx 376,99 \text{ cm}^3$$

$$b) \quad G = 100 \text{ cm}^2$$

$$\pi r^2 = 100$$

$$r^2 = \frac{100}{\pi}$$

$$r = \sqrt{\frac{100}{\pi}}$$

$$r \approx 5,64 \text{ cm}$$

$$r^2 + h^2 = m^2$$

$$5,64^2 + h^2 = 10^2$$

$$31,8096 + h^2 = 100$$

$$h^2 = 68,1904$$

$$h = 8,26 \text{ cm}$$

$$M = \pi r m$$

$$= \pi \cdot 5,64 \cdot 10 = 56,4\pi = 177,19 \text{ cm}^2$$

$$O = G + M = 277,19 \text{ cm}^2$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \cdot 5,64^2 \cdot 8,26$$

$$= 87,582 \dots \pi$$

$$\approx 275,15 \text{ cm}^3$$

$$c) \quad r^2 + h^2 = m^2$$

$$r^2 + 12^2 = 16^2$$

$$r^2 + 144 = 256$$

$$r^2 = 112$$

$$r = 10,58 \text{ cm}$$

$$G = \pi r^2 = \pi \cdot 10,58^2 = 111,9364\pi$$

$$\approx 351,66 \text{ cm}^2$$

$$M = \pi r m = \pi \cdot 10,58 \cdot 16 = 169,28\pi$$

$$\approx 531,81 \text{ cm}^2$$

$$O = G + M = 884,47 \text{ cm}^2$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \cdot 10,58^2 \cdot 12 = 447,7456\pi$$

$$\approx 1406,63 \text{ cm}^3$$



$$d) V = \frac{1}{3} \pi r^2 \cdot h$$

$$1000 = \frac{1}{3} \pi \cdot 4^2 \cdot h \quad | \cdot 3$$

$$3000 = \pi \cdot 16 \cdot h \quad | : 16\pi$$

$$\frac{3000}{16\pi} = h$$

$$59,68 \text{ cm} = h$$

$$r^2 + h^2 = m^2$$

$$16 + 59,68^2 = m^2$$

$$3577,7024 = m^2$$

$$59,81 \text{ cm} = m$$

$$G = \pi r^2 = \pi \cdot 4^2 = 16\pi$$

$$= 50,27 \text{ cm}^2$$

$$M = \pi r \cdot m = \pi \cdot 4 \cdot 59,81$$

$$= 239,24\pi$$

$$\approx 751,59 \text{ cm}^2$$

$$O = G + M = 801,86 \text{ cm}^2$$

$$3a) \left(\frac{a-b}{2}\right)^2 + h_p^2 = h_T^2$$

$$\left(\frac{8-4}{2}\right)^2 + h_p^2 = 10^2$$

$$2^2 + h_p^2 = 10^2$$

$$4 + h_p^2 = 100$$

$$h_p^2 = 96$$

$$h_p \approx 9,8 \text{ cm}$$

$$G = a^2 = 64 \text{ cm}^2$$

$$S = b^2 = 16 \text{ cm}^2$$

$$M = 4 \cdot \frac{1}{2} \cdot (a+b) \cdot h_T$$

$$= 4 \cdot \frac{1}{2} \cdot (8+4) \cdot 10$$

$$= 240 \text{ cm}^2$$

$$O = G + S + M$$

$$= 320 \text{ cm}^2$$

$$V = \frac{1}{3} \cdot h_p \cdot (a^2 + ab + b^2)$$

$$= \frac{1}{3} \cdot 9,8 \cdot (64 + 32 + 16)$$

$$= 365,87 \text{ cm}^3$$

$$b) S = 16$$

$$b^2 = 16$$

$$b = 4 \text{ cm}$$

$$\left(\frac{a-b}{2}\right)^2 + h_p^2 = h_T^2$$

$$\left(\frac{5-4}{2}\right)^2 + 10^2 = h_T^2$$

$$0,25 + 100 = h_T^2$$

$$100,25 = h_T^2$$

$$10,01 \text{ cm} = h_T$$

$$\begin{aligned}
 M &= 4 \cdot \frac{1}{2} \cdot (a+b) \cdot h_T \\
 &= 4 \cdot \frac{1}{2} \cdot (5+4) \cdot 10,01 \\
 &= 180,18 \text{ cm}^2
 \end{aligned}$$

$$G = S^2 = 25 \text{ cm}^2$$

$$O = G + S + M = 221,18 \text{ cm}^2$$

$$\begin{aligned}
 V &= \frac{1}{3} \cdot h_p \cdot (a^2 + ab + b^2) \\
 &= \frac{1}{3} \cdot 10 \cdot (25 + 9 + 16) \\
 &= 166,67 \text{ cm}^3
 \end{aligned}$$

$$\begin{aligned}
 4a) \quad h_K^2 + \left(\frac{r_1 - r_2}{2}\right)^2 &= m^2 \\
 16^2 + \left(\frac{10-6}{2}\right)^2 &= m^2 \\
 256 + 4 &= m^2 \\
 260 &= m^2 \\
 16,12 \text{ cm} &= m
 \end{aligned}$$

$$\begin{aligned}
 G &= \pi r_1^2 = \pi \cdot 10^2 \\
 &= 100\pi \\
 &= 314,16 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 S &= \pi r_2^2 = \pi \cdot 6^2 \\
 &= 36\pi \\
 &= 113,1 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 M &= (r_1 + r_2) \cdot \pi \cdot m \\
 &= (10 + 6) \cdot \pi \cdot 16,12 \\
 &= 257,92 \pi \\
 &= 810,3 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 O &= G + S + M \\
 &= 1237,56 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 V &= \frac{1}{3} \pi \cdot h_K \cdot (r_1^2 + r_1 r_2 + r_2^2) \\
 &= \frac{1}{3} \pi \cdot 16 \cdot (100 + 60 + 36) \\
 &= 1041,3 \pi \\
 &= 3284,01 \text{ cm}^3
 \end{aligned}$$

$$b) \quad h_K^2 + \left(\frac{r_1 - r_2}{2}\right)^2 = m^2$$

$$h_K^2 + \left(\frac{12-2}{2}\right)^2 = 16^2$$

$$h_K^2 + 25 = 256$$

$$h_K^2 = 231$$

$$h_K = 15,2 \text{ cm}$$

$$G = \pi r_1^2 = 144\pi$$

$$\approx 452,39 \text{ cm}^2$$

$$S = \pi r_2^2 = 4\pi$$

$$\approx 12,57 \text{ cm}^2$$

$$M = (r_1 + r_2) \cdot \pi \cdot m$$

$$= (12+2) \cdot \pi \cdot 16$$

$$= 224\pi$$

$$\approx 703,72 \text{ cm}^2$$

$$O = G + S + M$$

$$= 1.168,68 \text{ cm}^2$$

$$V = \frac{1}{3} \pi \cdot h_K \cdot (r_1^2 + r_1 r_2 + r_2^2)$$

$$= \frac{1}{3} \pi \cdot 15,2 \cdot (144 + 24 + 4)$$

$$= 871,46 \pi$$

$$\approx 2737,79 \text{ cm}^3$$