LÖSUNGEN (TEIL A)

Aufgabe 1

a) $x^2 + 2x - 8 = 0$

$x = -1 \pm \sqrt{1+8}$

$x = -1 \pm \sqrt{9}$

$x = -1 \pm 3$

$x_1 = -4$

$x_2 = 2$

b) $2x^2 - 8x - 24 = 0 \quad | :2$

$x^2 - 4x - 12 = 0$

$x = 2 \pm \sqrt{4+12}$

$x = 2 \pm \sqrt{16}$

$x = 2 \pm 4$

$x_1 = -2$

$x_2 = 6$

c) $x^2 + 4x = 0$

$x \cdot (x+4) = 0$

$x_1 = 0 \qquad x+4 = 0$

$x_2 = -4$

d) $x^3 + 4x^2 = 0$

$x^2 \cdot (x+4) = 0$

$x^2 = 0 \qquad x+4 = 0$

$x_1 = 0 \qquad x_2 = -4$

e) $x^4 - 10x^2 + 9 = 0 \quad | x^2 = z$

$z^2 - 10z + 9 = 0$

$z = 5 \pm \sqrt{25-9}$

$z = 5 \pm \sqrt{16}$

$z = 5 \pm 4$

$z_1 = 1 \qquad z_2 = 9 \quad | z = x^2$

$x^2 = 1 \quad | \sqrt{\ } \qquad x^2 = 9 \quad | \sqrt{\ }$

$x_1 = 1 \qquad x_3 = 3$

$x_2 = -1 \qquad x_4 = -3$

f) $x^5 - 8x^3 + 16x = 0$

$x \cdot (x^4 - 8x^2 + 16) = 0$

$x_1 = 0$ $\qquad x^4 - 8x^2 + 16 = 0 \quad | \, x^2 = z$

$z^2 - 8z + 16 = 0$

$z = 4 \pm \sqrt{16 - 16}$

$z = 4 \quad | \, z = x^2$

$x^2 = 4 \quad | \sqrt{\ }$

$x_2 = 2$

$x_3 = -2$

g) $x^3 - 27 = 0 \quad | +27$

$x^3 = 27 \quad | \sqrt[3]{\ }$

$x = 3$

Aufgabe 2

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 1 & 2 & 3 & 8 \\ 2 & -1 & -2 & -2 \end{array}\right) \begin{array}{l} \\ I - II \\ 2 \cdot I - III \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & -4 \\ 0 & 3 & 4 & 10 \end{array}\right) \begin{array}{l} \\ \\ 3 \cdot II + III \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & -1 & -2 & -4 \\ 0 & 0 & -2 & -2 \end{array}\right)$$

$\Rightarrow -2z = -2$

$z = 1$

$\Rightarrow -y - 2 = -4$

$-y = -2$

$y = 2$

$\Rightarrow x + 2 + 1 = 4$

$x = 1$

Aufgabe 3

a) $f'(x) = 6x + 4 \qquad f''(x) = 6$

b) $F_1(x) = x^3 + 2x^2 + x$

$F_2(x) = x^3 + 2x^2 + x + 2$

c) $A(1|1)$ auf $F \Rightarrow F(1) = 1$

$F(x) = x^3 + 2x^2 + x + c$

$F(1) = 1^3 + 2 \cdot 1^2 + 1 + c = 1$

$1 + 2 + 1 + c = 1$

$4 + c = 1$

$c = -3$

$\Rightarrow F(x) = x^3 + 2x^2 + x - 3$

Aufgabe 4

a) $\int_2^3 2x + 3\,dx = \left[x^2 + 3x\right]_2^3$

$= 3^2 + 3 \cdot 3 - (2^2 + 3 \cdot 2)$

$= 9 + 9 - (4 + 6)$

$= 18 - 10$

$= 8$

b) $\int_1^4 6x^2 + 2x\,dx = \left[2x^3 + x^2\right]_1^4 = 2 \cdot 4^3 + 4^2 - (2 \cdot 1^3 + 1^2)$

$= 128 + 16 - (2 + 1)$

$= 144 - 3$

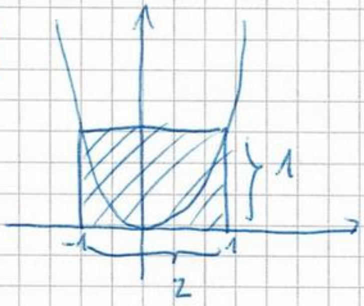
$= 141$

c) $\int_1^2 \frac{1}{x^2}\,dx = \int_1^2 x^{-2}\,dx = \left[-x^{-1}\right]_1^2 = \left[-\frac{1}{x}\right]_1^2$

$= -\frac{1}{2} - \left(-\frac{1}{1}\right) = -\frac{1}{2} + 1 = \frac{1}{2}$

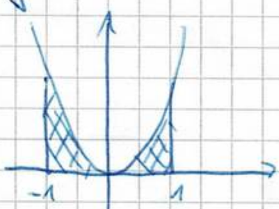
Aufgabe 5

a)



$A_{///} = 2 \cdot 1 = 2$ FE

$f(1) = 1^2 = 1$



$A_{\#} = \int_{-1}^{1} x^2\,dx = \left[\frac{1}{3}x^3\right]_{-1}^{1}$

$= \frac{1}{3} - \left(-\frac{1}{3}\right) = \frac{2}{3}$ FE

$\Rightarrow A_{gesucht} = A_{///} - A_{\#} = 2 - \frac{2}{3} = \underline{\underline{\frac{4}{3}}}$ FE

b) Schnittpunkte:

$x^2 = 2x$

$x^2 - 2x = 0$

$x \cdot (x-2) = 0$

$x_1 = 0 \quad x_2 = 2$

$f(0) = 0$

$f(2) = 2^2 = 4$

$S_1(0/0)$

$S_2(2/4)$

$$A = \int_0^2 g(x) - f(x)\, dx$$

$$= \int_0^2 2x - x^2\, dx = \left[x^2 - \frac{1}{3}x^3\right]_0^2 = 2^2 - \frac{1}{3}\cdot 2^3 - \left(0^2 - \frac{1}{3}\cdot 0^3\right)$$

$$= 4 - \frac{8}{3} - 0$$

$$= \frac{12}{3} - \frac{8}{3}$$

$$= \frac{4}{3} \text{ FE}$$

Aufgabe 6

a) $-2x^2 + 4x = 0$

$x \cdot (-2x + 4) = 0$

$x_1 = 0$ $\qquad -2x + 4 = 0$

$-2x = -4$

$x_2 = 2$

b) $$\int_0^a f(x)\, dx = \int_0^a -2x^2 + 4x\, dx = \left[-\frac{2}{3}x^3 + 2x^2\right]_0^a$$

$$= -\frac{2}{3}a^3 + 2a^2 - 0$$

$$= -\frac{2}{3}a^3 + 2a^2$$

Es gilt: $\int_0^a f(x)\, dx = 0$

$\Rightarrow -\frac{2}{3}a^3 + 2a^2 = 0$

$$a^2 \cdot \left(-\frac{2}{3}a + 2\right) = 0$$

$$a^2 = 0 \qquad -\frac{2}{3}a + 2 = 0$$

$$a_1 = 0 \qquad -\frac{2}{3}a = -2$$

$$a = -2 \cdot \left(-\frac{3}{2}\right) = 3$$

$$a_2 = 3$$

Aufgabe 7

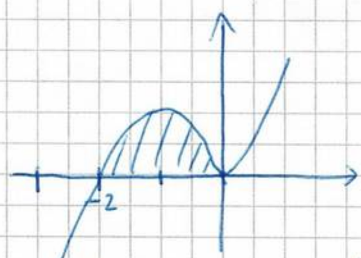
a) $x^3 + 2x^2 = 0$

$$x^2 \cdot (x+2) = 0$$

$$x^2 = 0 \qquad x + 2 = 0$$

$$x_1 = 0 \qquad x_2 = -2$$

b)



$$A = \int_{-2}^{0} f(x)\,dx = \int_{-2}^{0} x^3 + 2x^2\,dx$$

$$= \left[\frac{1}{4}x^4 + \frac{2}{3}x^3\right]_{-2}^{0}$$

$$= 0 - \left(\frac{1}{4} \cdot (-2)^4 + \frac{2}{3} \cdot (-2)^3\right)$$

$$= -\left(4 - \frac{16}{3}\right)$$

$$= -\left(\frac{12}{3} - \frac{16}{3}\right)$$

$$= -\left(-\frac{4}{3}\right)$$

$$\underline{\underline{= 4/3 \text{ FE}}}$$

Aufgabe 8

$x^4 = 4 + 3x^2 \quad | -3x^2 \quad | -4$

$x^4 - 3x^2 - 4 = 0 \quad | \, x^2 = z$

$z^2 - 3z - 4 = 0$

$z = 1{,}5 \pm \sqrt{2{,}25 + 4}$

$z = 1{,}5 \pm \sqrt{6{,}25}$

$z = 1{,}5 \pm 2{,}5$

$z_1 = -1 \qquad z_2 = 4 \quad | \, z = x^2$

$x^2 = -1 \qquad x^2 = 4 \quad | \sqrt{\ }$

$\lightning \qquad x_1 = 2$

$x_2 = -2$

Aufgabe 9

a) $\int_0^1 f(x)\,dx = \int_0^1 -6x^2 + 12x + 18\,dx$

$= \left[-2x^3 + 6x^2 + 18x\right]_0^1$

$= -2 + 6 + 18 - (0)$

$= 22$

b) $H(1|24)$

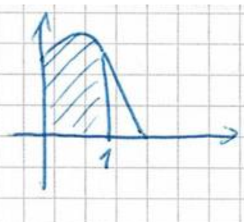
$g(x) = -57{,}6x + b$

$H(1|24)$ auf $g \Rightarrow g(1) = 24$

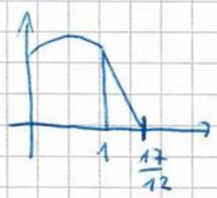
$-57{,}6 + b = 24$

$b = 81{,}6$

$\Rightarrow g(x) = -57{,}6x + 81{,}6$

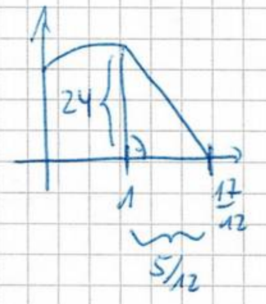


$$A_{///} = \int_0^1 f(x)\,dx = 22 \text{ FE}$$



Nullstelle von g:

$$-57{,}6x + 81{,}6 = 0$$
$$-57{,}6x = -81{,}6$$
$$x = \frac{816}{576} = \frac{17}{12}$$



$$A_{\text{Dreieck}} = \frac{1}{2} \cdot 24 \cdot \frac{5}{12} = 5$$

$$A_{\text{links von } g} = A_{///} + A_{\text{Dreieck}} = 22 + 5 = 27 \text{ FE}$$

$\Rightarrow$ 27 ist die Hälfte von 54 ✓

<u>Aufgabe 10</u>

a) $F(x) = -x^3 + 3x^2$

b) Nullstelle: $-3x^2 + 6x = 0$

$$x \cdot (-3x + 6) = 0$$
$$x_1 = 0 \qquad -3x + 6 = 0$$
$$-3x = -6$$
$$x_2 = 2$$

$$A_{\text{links}} = \int_0^2 -3x^2 + 6x \, dx$$

$$= \left[-x^3 + 3x^2\right]_0^2 = -2^3 + 3 \cdot 2^2 - 0$$
$$= -8 + 12 = 4 \text{ FE}$$

Fläche rechts: $\int_2^3 -3x^2 + 6x \, dx = \left[-x^3 + 3x^2\right]_2^3$

$$= -3^3 + 3 \cdot 3^2 - \left(-2^3 + 3 \cdot 2^2\right)$$
$$= -27 + 27 - 4$$
$$= -4$$

$\Rightarrow A_{\text{rechts}} = 4 \text{ FE}$

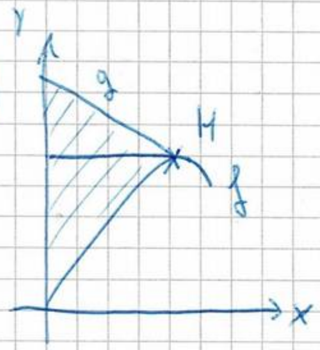
$\Rightarrow A = A_{\text{links}} + A_{\text{rechts}} = \underline{\underline{8 \text{ FE}}}$

Aufgabe 11

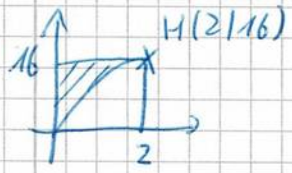
a) $A = \int_0^2 f(x) \, dx = \int_0^2 -x^3 + 12x \, dx$

$$= \left[-\frac{1}{4}x^4 + 6x^2\right]_0^2 = -\frac{1}{4} \cdot 2^4 + 6 \cdot 2^2 - 0$$
$$= -4 + 24$$
$$= 20 \text{ FE}$$

b)



Wir zerlegen die Fläche in zwei Teile:

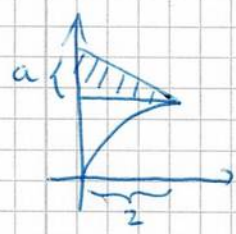


$$A_{\square} = 2 \cdot 16 = 32 \text{ FE}$$

$$A_{\triangle} = \int_0^2 f(x)\,dx = 20 \quad \text{(Aufgabenteil a)}$$

$$A_{///} = A_{\square} - A_{\triangle} = 32 - 20 = 12$$

⇒ Der obere Teil der schraffierten Fläche muss $20 - 12 = 8$ FE groß sein.

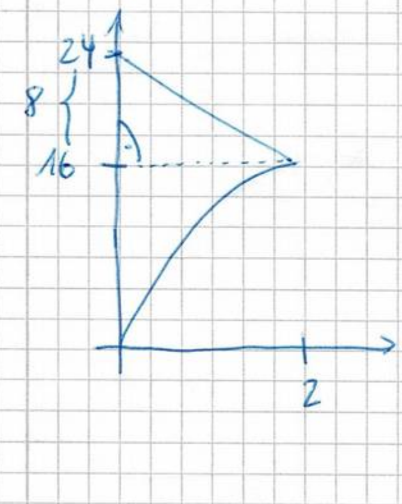


Der obere Teil ist aber ein rechtwinkliges Dreieck mit Breite 2

$$\Rightarrow \quad \frac{1}{2} \cdot 2 \cdot a = 8$$

$$a = 8$$

⇒ Der Schnittpunkt liegt bei $16 + 8 = 24$



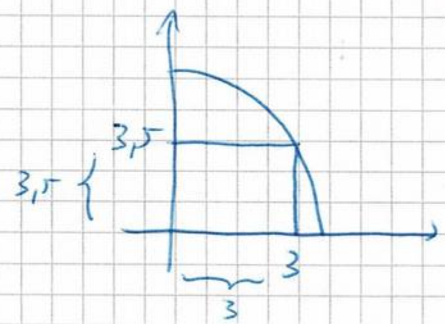
Aufgabe 12

a) $f(x) = \frac{1}{2}(16 - x^2) = 8 - 0{,}5x^2 = -0{,}5x^2 + 8$

$P(3 \mid f(3)) \rightarrow$ Was ist der y-Wert?

$f(3) = -0{,}5 \cdot 3^2 + 8 = -0{,}5 \cdot 9 + 8$

$= -4{,}5 + 8 = 3{,}5$



$\Rightarrow U = 3 + 3{,}5 + 3 + 3{,}5 = 13$ LE

b) $U(x) = 2x + 2 \cdot f(x) = 2x + 2 \cdot (-0{,}5x^2 + 8)$

$= 2x - x^2 + 16$

$= -x^2 + 2x + 16$

$U'(x) = -2x + 2$

Notw. Bed.: $U'(x) = 0$

$-2x + 2 = 0$

$-2x = -2$

$\underline{x = 1}$