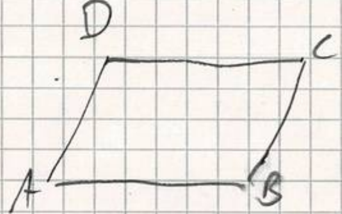
LÖSUNGEN

Teil A: Hilfsmittelfreier Teil

1a)



$$\vec{AB} = \begin{pmatrix} 3-2 \\ 5-1 \\ 2-1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$$

$$\vec{DC} = \begin{pmatrix} 0-(-1) \\ 5-1 \\ 4-3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$$

$\vec{AB} = \vec{DC}$

$\Rightarrow$ ABCD Parallelogramm

b) $\vec{AB} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$

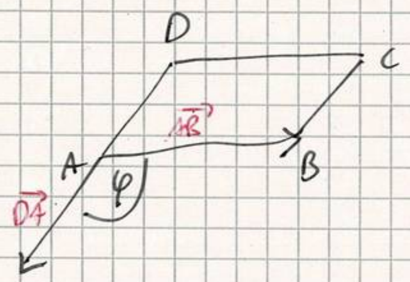
$$\vec{AD} = \begin{pmatrix} -1-2 \\ 1-1 \\ 3-1 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$$

$$\vec{AB} \circ \vec{AD} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} \circ \begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix} = -3 + 2 = -1 \neq 0$$

$\Rightarrow$ kein rechter Winkel

$\Rightarrow$ kein Rechteck

c)



2) $$\left(\begin{pmatrix}a\\1\\2\end{pmatrix}\times\begin{pmatrix}1\\2\\1\end{pmatrix}\right)\circ\begin{pmatrix}2\\1\\2\end{pmatrix}=10$$

$$\begin{array}{cc} a & 1 \\ 1 & 2 \\ 2 & 1 \\ a & 1 \\ 1 & 2 \end{array}$$

$$\begin{pmatrix}1-4\\2-a\\2a-1\end{pmatrix}\circ\begin{pmatrix}2\\1\\2\end{pmatrix}=10$$

$$\begin{pmatrix}-3\\2-a\\2a-1\end{pmatrix}\circ\begin{pmatrix}2\\1\\2\end{pmatrix}=10$$

$$-6+2-a+4a-2=10$$

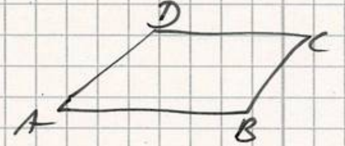
$$-6+3a=10 \quad |+6$$

$$3a=16 \quad |:3$$

$$a=\frac{16}{3}$$

Teil B: Teil mit Hilfsmitteln

1a) $\vec{AB} = \begin{pmatrix}7-1\\2-0\\6-2\end{pmatrix} = \begin{pmatrix}6\\2\\4\end{pmatrix}$



$\vec{DC} = \begin{pmatrix}5-2\\2-1\\5-3\end{pmatrix} = \begin{pmatrix}3\\1\\2\end{pmatrix}$

$\vec{AD} = \begin{pmatrix}2-1\\1-0\\3-2\end{pmatrix} = \begin{pmatrix}1\\1\\1\end{pmatrix}$

$\vec{BC} = \begin{pmatrix}5-7\\2-2\\5-6\end{pmatrix} = \begin{pmatrix}-2\\0\\-1\end{pmatrix}$

$\vec{AB}$ und $\vec{DC}$ sind Vielfache voneinander: $\vec{AB} = 2 \cdot \vec{DC}$
$\Rightarrow$ $\vec{AB}$ parallel zu $\vec{DC}$

$\vec{AD}$ und $\vec{BC}$ keine Vielfache voneinander
$\Rightarrow$ $\vec{AD}$ nicht parallel zu $\vec{BC}$

$\Rightarrow$ ABCD ist ein Trapez

$\vec{AB} \neq \vec{DC}$
$\Rightarrow$ ABCD kein Parallelogramm

$|\vec{AD}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$
$|\vec{BC}| = \sqrt{(-2)^2+1^2} = \sqrt{5}$
$|\vec{AD}| \neq |\vec{BC}|$
$\Rightarrow$ kein gleichschenkliges Trapez

b)

$\cos\alpha = \frac{\vec{AB} \circ \vec{AD}}{|\vec{AB}| \cdot |\vec{AD}|} = \frac{\begin{pmatrix}6\\2\\4\end{pmatrix} \circ \begin{pmatrix}1\\1\\1\end{pmatrix}}{\sqrt{56} \cdot \sqrt{3}} = \frac{12}{\sqrt{168}}$

$|\vec{AB}| = \sqrt{6^2+2^2+4^2} = \sqrt{36+4+16} = \sqrt{56}$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{12}{\sqrt{168}}\right) \approx 22{,}21°$$

$$\cos\beta = \frac{\overrightarrow{BA} \circ \overrightarrow{BC}}{|\overrightarrow{BA}| \cdot |\overrightarrow{BC}|} = \frac{\begin{pmatrix}-6\\-2\\-4\end{pmatrix} \circ \begin{pmatrix}-2\\0\\-1\end{pmatrix}}{\sqrt{56} \cdot \sqrt{5}} = \frac{16}{\sqrt{280}}$$

$$\Rightarrow \beta = \cos^{-1}\left(\frac{16}{\sqrt{280}}\right) \approx 17{,}02°$$

$$\cos\gamma = \frac{\overrightarrow{CB} \circ \overrightarrow{CD}}{|\overrightarrow{CB}| \cdot |\overrightarrow{CD}|} = \frac{\begin{pmatrix}2\\0\\1\end{pmatrix} \circ \begin{pmatrix}-3\\-1\\-2\end{pmatrix}}{\sqrt{5} \cdot \sqrt{14}} = \frac{-8}{\sqrt{70}}$$

$$|\overrightarrow{CD}| = \sqrt{9+1+4} = \sqrt{14}$$

$$\Rightarrow \gamma = \cos^{-1}\left(\frac{-8}{\sqrt{70}}\right) \approx 162{,}98°$$

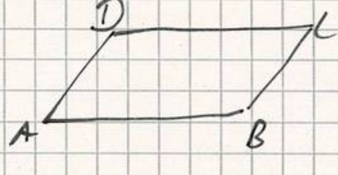
$$\cos\delta = \frac{\overrightarrow{DA} \circ \overrightarrow{DC}}{|\overrightarrow{DA}| \cdot |\overrightarrow{DC}|} = \frac{\begin{pmatrix}-1\\-1\\-1\end{pmatrix} \circ \begin{pmatrix}3\\1\\2\end{pmatrix}}{\sqrt{3} \cdot \sqrt{14}} = \frac{-6}{\sqrt{42}}$$

$$\Rightarrow \delta = \cos^{-1}\left(\frac{-6}{\sqrt{42}}\right) \approx 157{,}79°$$

oder: $\delta = 360° - \alpha - \beta - \gamma$

c) $$\overrightarrow{OD_{neu}} = \overrightarrow{OC} + \overrightarrow{BA} = \begin{pmatrix}5\\2\\5\end{pmatrix} + \begin{pmatrix}-6\\-2\\-4\end{pmatrix} = \begin{pmatrix}-1\\0\\1\end{pmatrix}$$

$$\Rightarrow D_{neu}(-1/0/1)$$



d) Distanz zu A: $\overrightarrow{EA} = \begin{pmatrix} 1-6 \\ 0-1 \\ 2-z \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ 2-z \end{pmatrix}$

Distanz $\quad |\overrightarrow{EA}| = \sqrt{(-5)^2 + (-1)^2 + (2-z)^2}$

$= \sqrt{25 + 1 + 4 - 4z + z^2}$

$= \sqrt{z^2 - 4z + 30}$

Distanz zu B: $\overrightarrow{EB} = \begin{pmatrix} 7-6 \\ 2-1 \\ 6-z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 6-z \end{pmatrix}$

$|\overrightarrow{EB}| = \sqrt{1^2 + 1^2 + (6-z)^2}$

$= \sqrt{1 + 1 + 36 - 12z + z^2}$

$= \sqrt{z^2 - 12z + 38}$

gleiche Entfernung:

$$|\overrightarrow{EA}| = |\overrightarrow{EB}|$$

$$\begin{aligned}
\sqrt{z^2 - 4z + 30} &= \sqrt{z^2 - 12z + 38} && |\,(\;)^2 \\
z^2 - 4z + 30 &= z^2 - 12z + 38 && |\,-z^2 \\
-4z + 30 &= -12z + 38 && |\,-30 \\
-4z &= -12z + 8 && |\,+12z \\
8z &= 8 \\
z &= 1
\end{aligned}$$

e) ① $|\overrightarrow{FA}| = |\overrightarrow{FB}|$

$\left|\begin{pmatrix} 1-1 \\ 0-(18-2a) \\ 2-a \end{pmatrix}\right| = \left|\begin{pmatrix} 7-1 \\ 2-(18-2a) \\ 6-a \end{pmatrix}\right|$

$\left|\begin{pmatrix} 0 \\ -18+2a \\ 2-a \end{pmatrix}\right| = \left|\begin{pmatrix} 6 \\ -16+2a \\ 6-a \end{pmatrix}\right|$

$\sqrt{(-18+2a)^2 + (2-a)^2} = \sqrt{36 + (-16+2a)^2 + (6-a)^2}$

$$\sqrt{324-72a+4a^2+4-4a+a^2} = \sqrt{36+256-64a+4a^2+36-12a+a^2}$$
$$\sqrt{5a^2-76a+328} = \sqrt{5a^2-76a+328} \checkmark$$

⑪ Abstand $|\vec{FA}|$:

$$|\vec{FA}| = \sqrt{5a^2-76a+328} = f(a)$$

gesucht: Minimum von f

f hat genau dann ein Minimum, wenn $g(x) = (f(x))^2$ eines hat:

$$g(x) = 5a^2 - 76a + 328$$
$$g'(x) = 10a - 76$$
$$g''(x) = 10$$

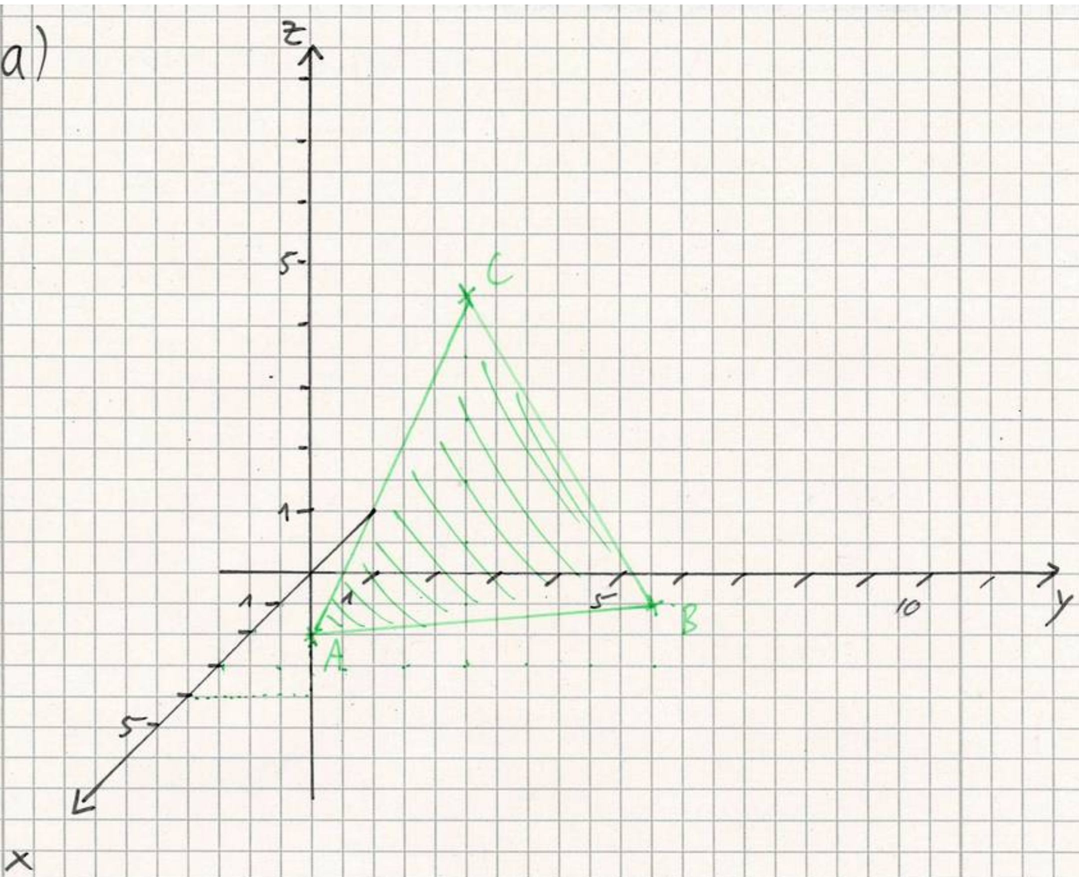
Notw. Bed.: $g'(x) = 0$
$$10a - 76 = 0$$
$$10a = 76$$
$$a = 7{,}6$$

Hinr. Bed.: $g'(x)$ und $g''(x) \neq 0$
$$g''(7{,}6) = 10 > 0 \Rightarrow \text{Min. bei } a = 7{,}6$$

$\Rightarrow a = 7{,}6$

(Ränder nicht vorhanden, bzw. bei einer quadr. Funktion muss die Stelle beim Scheitelpunkt liegen)

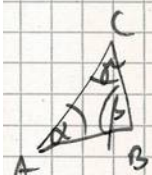
2a)



b) $\vec{AB} = \begin{pmatrix} 3-4 \\ 7-2 \\ 1-1 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix}$ $\quad |\vec{AB}| = \sqrt{1^2+5^2} = \sqrt{26}$

$\vec{AC} = \begin{pmatrix} 3-4 \\ 4-2 \\ 6-1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$ $\quad |\vec{AC}| = \sqrt{1^2+2^2+5^2} = \sqrt{30}$

$\vec{BC} = \begin{pmatrix} 3-3 \\ 4-7 \\ 6-1 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 5 \end{pmatrix}$ $\quad |\vec{BC}| = \sqrt{3^2+5^2} = \sqrt{34}$



$$\cos\alpha = \frac{\vec{AB} \circ \vec{AC}}{|\vec{AB}| \cdot |\vec{AC}|} = \frac{\begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} \circ \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}}{\sqrt{26} \cdot \sqrt{30}} = \frac{11}{\sqrt{780}}$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{11}{\sqrt{780}}\right) \approx 66{,}8°$$

$$\cos\beta = \frac{\vec{BA} \circ \vec{BC}}{|\vec{BA}| \cdot |\vec{BC}|} = \frac{\begin{pmatrix} 1 \\ -5 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 0 \\ -3 \\ 5 \end{pmatrix}}{\sqrt{26} \cdot \sqrt{34}} = \frac{15}{\sqrt{884}}$$

$$\Rightarrow \beta = \cos^{-1}\left(\frac{15}{\sqrt{884}}\right) \approx 59{,}7°$$

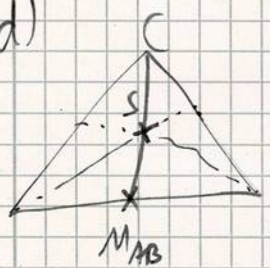
$$\cos \gamma = \frac{\vec{CA} \cdot \vec{CB}}{|\vec{CA}| \cdot |\vec{CB}|} = \frac{\begin{pmatrix}1\\-2\\-5\end{pmatrix} \circ \begin{pmatrix}0\\3\\-5\end{pmatrix}}{\sqrt{30} \cdot \sqrt{34}} = \frac{19}{\sqrt{1020}}$$

$$\Rightarrow \gamma = \cos^{-1}\left(\frac{19}{\sqrt{1020}}\right) = 53{,}49°$$

oder: $\gamma = 180° - \alpha - \beta$

c) Seiten alle verschieden lang
Kein rechter Winkel
$\Rightarrow$ Kein besonderes Dreieck

d)



$$\vec{OM_{AB}} = \vec{OA} + \frac{1}{2} \cdot \vec{AB} = \begin{pmatrix}4\\2\\1\end{pmatrix} + \frac{1}{2} \cdot \begin{pmatrix}-1\\5\\0\end{pmatrix} = \begin{pmatrix}3{,}5\\4{,}5\\1\end{pmatrix}$$

$$\vec{OS} = \vec{OM_{AB}} + \frac{1}{3} \cdot \vec{M_{AB}C}$$

$$= \begin{pmatrix}3{,}5\\4{,}5\\1\end{pmatrix} + \frac{1}{3} \cdot \begin{pmatrix}3-3{,}5\\4-4{,}5\\6-1\end{pmatrix}$$

$$= \begin{pmatrix}3{,}5\\4{,}5\\1\end{pmatrix} + \frac{1}{3} \cdot \begin{pmatrix}-0{,}5\\-0{,}5\\5\end{pmatrix}$$

$$= \begin{pmatrix}7/2\\9/2\\1\end{pmatrix} + \begin{pmatrix}-1/6\\-1/6\\5/3\end{pmatrix}$$

$$= \begin{pmatrix}10/3\\13/3\\8/3\end{pmatrix}$$

$$\Rightarrow S(10/3 \mid 13/3 \mid 8/3)$$

e)

$$A = \frac{1}{2} \cdot |\vec{AB} \times \vec{AC}| = \frac{1}{2} \cdot \left|\begin{pmatrix}-1\\5\\0\end{pmatrix} \times \begin{pmatrix}-1\\2\\5\end{pmatrix}\right| = \frac{1}{2} \cdot \left|\begin{pmatrix}25\\5\\3\end{pmatrix}\right|$$

$$\begin{array}{cc} -1 & -1 \\ 5 & 2 \\ 0 & 5 \\ -1 & -1 \\ 5 & 2 \end{array}$$

$= \frac{1}{2} \cdot \sqrt{25^2 + 5^2 + 3^2}$

$= \frac{1}{2} \cdot \sqrt{659}$

$\approx 12,84$ FE

f) I $E: \vec{x} = \vec{OA} + r \cdot \vec{AB} + s \cdot \vec{AC}$

$= \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + r \cdot \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$

$\begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 25 \\ 5 \\ 3 \end{pmatrix}$

$\Rightarrow E: 25x + 5y + 3z = d$

$\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} \circ \begin{pmatrix} 25 \\ 5 \\ 3 \end{pmatrix} = 100 + 10 + 3 = 113$

$\Rightarrow E: 25x + 5y + 3z = 113$

II $g: \vec{x} = \begin{pmatrix} 1+r \\ 7+2r \\ 5+r \end{pmatrix}$

Einsetzen in E:

$25 \cdot (1+r) + 5 \cdot (7+2r) + 3(5+r) = 113$

$25 + 25r + 35 + 10r + 15 + 3r = 113$

$75 + 38r = 113 \quad | -75$

$38r = 38$

$r = 1$

$\Rightarrow \vec{OS} = \begin{pmatrix} 1+1 \\ 7+2 \\ 5+1 \end{pmatrix} = \begin{pmatrix} 2 \\ 9 \\ 6 \end{pmatrix}$

III

$\begin{pmatrix} 2 \\ 9 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} + r \cdot \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \quad \Big| - \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$

$\begin{pmatrix} -2 \\ 7 \\ 5 \end{pmatrix} = r \cdot \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$

$$\left(\begin{array}{cc|c} -1 & -1 & -2 \\ 5 & 2 & 7 \\ 0 & 5 & 5 \end{array}\right)$$

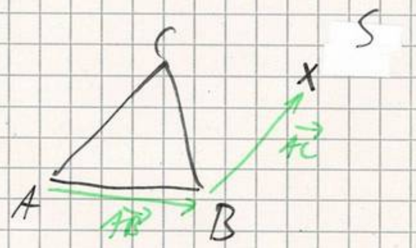
$\Rightarrow 5s = 5$

$s = 1$

$\Rightarrow 5r + 2 = 7$ und $-r - 1 = -2$

$5r = 5$ $\quad -r = -1$

$r = 1$ $\quad r = 1$



Der Punkt liegt außerhalb