LÖSUNGEN

1) a) $\log_a(a^m) = m$, denn $a^m = a^m$

b) $\log_{a^2}(\sqrt{a}) = x \Rightarrow (a^2)^x = \sqrt{a}$

$a^{2x} = a^{\frac{1}{2}}$

$\Rightarrow x = \frac{1}{4}$

c) $\log_{\sqrt{a}}(a^3) = x \Rightarrow (\sqrt{a})^x = a^3$

$(a^{\frac{1}{2}})^x = a^3$

$a^{\frac{1}{2}x} = a^3$

$\Rightarrow x = 6$

2) a) $\log_2(\log_2(x)) = 2 \Rightarrow 2^2 = \log_2(x)$

$4 = \log_2(x)$

$\Rightarrow 2^4 = x$

$16 = x$

b) $\log_4(8x) = 2 \Rightarrow 4^2 = 8x$

$16 = 8x$

$2 = x$

c) $\log_x(4) = -0{,}5 \Rightarrow x^{-0{,}5} = 4$

$\frac{1}{\sqrt{x}} = 4$

$1 = 4\sqrt{x}$

$\frac{1}{4} = \sqrt{x} \quad |()^2$

$\frac{1}{16} = x$

d) $\log_4(\log_2(x)) = 2 \Rightarrow 4^2 = \log_2(x)$

$16 = \log_2(x)$

$\Rightarrow 2^{16} = x$

3) a) $\log_a\left(\frac{3ab}{b^2}\right) = \log_a(3ab) - \log_a(b^2)$

$= \log_a(3) + \log_a(a) + \log_a(b) - 2 \cdot \log_a(b)$

$= \log_a(3) + 1 + \log_a(b) - 2 \cdot \log_a(b)$

$= \log_a(3) - \log_a(b)$

b) $\log_a\left(\frac{b \cdot (b-c)}{(3b+c)^2}\right) = \log_a(b \cdot (b-c)) - \log_a((3b+c)^2)$

$= \log_a(b) + \log_a(b-c) - 2 \cdot \log_a(3b+c)$

c) $\log_a\left(\frac{3\sqrt{b} \cdot \frac{5}{c}}{6bc}\right) = \log_a(3\sqrt{b} \cdot \frac{5}{c}) - \log_a(6bc)$

$= \log_a(3\sqrt{b} \cdot \frac{5}{c}) - \log_a(6) - \log_a(b) - \log_a(c)$

$= \log_a(3\sqrt{b}) + \log_a(\frac{5}{c}) - \log_a(6) - \log_a(b) - \log_a(c)$

$= \log_a(3) + \frac{1}{2} \cdot \log_a(b) + \log_a(5) - \log_a(c) - \log_a(6) - \log_a(b) - \log_a(c)$

$= \log_a(3) - \frac{1}{2} \cdot \log_a(b) + \log_a(5) - 2\log_a(c) - \log_a(6)$

4 a) $\log_a(5) + \log_a(2) - 2 \cdot \log_a(7)$

$= \log_a(10) - \log_a(49)$

$= \log_a\left(\frac{10}{49}\right)$

b) $\left(\log_3(27) + 2 \cdot \log_3(9)\right) \cdot \frac{1}{7}$

$= \left(\log_3(27) + \log_3(81)\right) \cdot \frac{1}{7}$

$= \log_3(2187) \cdot \frac{1}{7}$

$= \log_3\left(2187^{\frac{1}{7}}\right)$

$= \log_3\left(\sqrt[7]{2187}\right)$

$= \log_3(3)$

$= 1$

c) $\left(\log_3(x^2-25) - \log_3(x-5)\right) \cdot 3$

$= \left(\log_3\left(\frac{x^2-25}{x-5}\right)\right) \cdot 3$

Binomische Regeln

$= \log_3(x+5) \cdot 3$

$= \log_3\left((x+5)^3\right)$

d) $\log_a(5) + \frac{\log_7(10)}{2 \cdot \log_7(a)}$

$= \log_a(5) + \frac{1}{2} \cdot \frac{\log_7(10)}{\log_7(a)}$

$= \log_a(5) + \frac{1}{2} \cdot \log_a(10)$

$= \log_a(5) + \log_a(\sqrt{10}) = \log_a(5 \cdot \sqrt{10})$

e) $\log_5(7) + \log_5(7)$

$= 2 \cdot \log_5(7)$

$= \log_5(49)$

5) a) $x + 7 = 2^4$

$x + 7 = 16$

$x = 9$

b) $\log_2(2x + 4) = 5$

$2^5 = 2x + 4$

$32 = 2x + 4$

$28 = 2x$

$14 = x$

c) $(\log_{10}(x))^2 + \log_{10}(x) = 6 \quad | \log_{10}(x) = z$

$z^2 + z = 6$

$z^2 + z - 6 = 0$

$z = -0{,}5 \pm \sqrt{6{,}25}$

$z = -0{,}5 \pm 2{,}5$

$z_1 = -3 \qquad z_2 = 2 \quad | z = \log_{10}(x)$

$\log_{10}(x) = -3 \qquad \log_{10}(x) = 2$

$x_1 = 10^{-3} \qquad x_2 = 10^2$

$x_1 = \frac{1}{1000} \qquad x_2 = 100$

d) $\log_3(2x-5)-\log_3(x-1)=3$

$$\log_3\left(\frac{2x-5}{x-1}\right)=3$$

$$\frac{2x-5}{x-1}=3^3$$

$$\frac{2x-5}{x-1}=27$$

$$2x-5=27(x-1)$$

$$2x-5=27x-27$$

$$22=25x$$

$$\frac{22}{25}=x$$

6 a) $\begin{pmatrix}0{,}8 & 0{,}2 & 0{,}1\\ 0{,}1 & 0{,}7 & 0{,}1\\ 0{,}1 & 0{,}1 & 0{,}8\end{pmatrix}\cdot\begin{pmatrix}2600\\2200\\5200\end{pmatrix}=\begin{pmatrix}2080+440+520\\260+1540+520\\260+220+4160\end{pmatrix}=\begin{pmatrix}3040\\2320\\4640\end{pmatrix}$

b) $\begin{pmatrix}0{,}8 & 0{,}2 & 0{,}1\\ 0{,}1 & 0{,}7 & 0{,}1\\ 0{,}1 & 0{,}1 & 0{,}8\end{pmatrix}\cdot\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}2600\\2200\\5200\end{pmatrix},$

$$\left(\begin{array}{ccc|c}0{,}8 & 0{,}2 & 0{,}1 & 2600\\ 0{,}1 & 0{,}7 & 0{,}1 & 2200\\ 0{,}1 & 0{,}1 & 0{,}8 & 5200\end{array}\right)\begin{array}{l}\\ 8\cdot\mathrm{II}-\mathrm{I}\\ 8\cdot\mathrm{III}-\mathrm{I}\end{array}$$

$$\left(\begin{array}{ccc|c}0{,}8 & 0{,}2 & 0{,}1 & 2600\\ 0 & 5{,}4 & 0{,}7 & 15.000\\ 0 & 0{,}6 & 6{,}3 & 39.000\end{array}\right)\begin{array}{l}\\ \\ 9\cdot\mathrm{III}-\mathrm{II}\end{array}$$

$$\left(\begin{array}{ccc|c}0{,}8 & 0{,}2 & 0{,}1 & 2600\\ 0 & 5{,}4 & 0{,}7 & 15.000\\ 0 & 0 & 56 & 336.000\end{array}\right)$$

$$\Rightarrow 56x_3=336.000$$
$$x_3=6000$$

$$\Rightarrow \quad 5{,}4x_2 + 0{,}7 \cdot 6000 = 15.000$$
$$5{,}4x_2 + 4200 = 15.000$$
$$5{,}4x_2 = 10.800$$
$$x_2 = 2000$$

$$\Rightarrow 0{,}8x_1 + 400 + 600 = 2600$$
$$0{,}8x_1 = 1600$$
$$x_1 = 2000$$

A: 2000
B: 2000
C: 6000

c) $$\begin{pmatrix} 0{,}8 & 0{,}2 & 0{,}1 \\ 0{,}1 & 0{,}7 & 0{,}1 \\ 0{,}1 & 0{,}1 & 0{,}8 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} 0{,}8x_1 + 0{,}2x_2 + 0{,}1x_3 \\ 0{,}1x_1 + 0{,}7x_2 + 0{,}1x_3 \\ 0{,}1x_1 + 0{,}1x_2 + 0{,}8x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} -0{,}2x_1 + 0{,}2x_2 + 0{,}1x_3 \\ 0{,}1x_1 - 0{,}3x_2 + 0{,}1x_3 \\ 0{,}1x_1 + 0{,}1x_2 - 0{,}2x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} -0{,}2 & 0{,}2 & 0{,}1 & 0 \\ 0{,}1 & -0{,}3 & 0{,}1 & 0 \\ 0{,}1 & 0{,}1 & -0{,}2 & 0 \end{array}\right) \begin{array}{l} \\ 2\cdot\text{II} + \text{I} \\ 2\cdot\text{III} + \text{I} \end{array}$$

$$\left(\begin{array}{ccc|c} -0{,}2 & 0{,}2 & 0{,}1 & 0 \\ 0 & -0{,}4 & 0{,}3 & 0 \\ 0 & 0{,}4 & -0{,}3 & 0 \end{array}\right) \begin{array}{l} \\ \\ \text{II} + \text{III} \end{array}$$

$$\left(\begin{array}{ccc|c} -0{,}2 & 0{,}2 & 0{,}1 & 0 \\ 0 & -0{,}4 & 0{,}3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$\Rightarrow x_3 = r, \quad r \in \mathbb{R}$

$\Rightarrow -0{,}4\, x_2 + 0{,}3\, r = 0$

$-0{,}4\, x_2 = -0{,}3\, r$

$x_2 = \frac{3}{4} r$

$\Rightarrow -0{,}2\, x_1 + 0{,}2 \cdot 0{,}75\, r + 0{,}1\, r = 0$

$-0{,}2\, x_1 + 0{,}15\, r + 0{,}1\, r = 0$

$-0{,}2\, x_1 = -0{,}25\, r$

$x_1 = \frac{5}{4} r$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5/4\, r \\ 3/4\, r \\ r \end{pmatrix}$$

Gesamtpopulation: $\frac{5}{4} r + \frac{3}{4} r + r = 3r$

$3r = 10000$

$r = 3333{,}\overline{3}$

$\Rightarrow$ A : $\frac{5}{4} \cdot 3333{,}\overline{3} = 4166{,}\overline{6} \approx 4167$ Personen

B : $\frac{3}{4} \cdot 3333{,}\overline{3} = 2500$ "

C : $3333{,}\overline{3} \approx 3333$ "

d)
$$\left(\begin{array}{ccc|ccc} 0{,}8 & 0{,}2 & 0{,}1 & 1 & 0 & 0 \\ 0{,}1 & 0{,}7 & 0{,}1 & 0 & 1 & 0 \\ 0{,}1 & 0{,}1 & 0{,}8 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ 8 \cdot II - I \\ 8 \cdot III - I \end{array}$$

$$\left(\begin{array}{ccc|ccc} 0{,}8 & 0{,}2 & 0{,}1 & 1 & 0 & 0 \\ 0 & 5{,}4 & 0{,}7 & -1 & 8 & 0 \\ 0 & 0{,}6 & 6{,}3 & -1 & 0 & 8 \end{array}\right) \begin{array}{l} \\ \\ 9 \cdot III - II \end{array}$$

d)

$$\begin{pmatrix} 0{,}8 & 0{,}2 & 0{,}1 \\ 0{,}1 & 0{,}7 & 0{,}1 \\ 0{,}1 & 0{,}1 & 0{,}8 \end{pmatrix} \cdot \begin{pmatrix} 0{,}8 & 0{,}2 & 0{,}1 \\ 0{,}1 & 0{,}7 & 0{,}1 \\ 0{,}1 & 0{,}1 & 0{,}8 \end{pmatrix}$$

$$= \begin{pmatrix} 0{,}67 & 0{,}31 & 0{,}18 \\ 0{,}16 & 0{,}52 & 0{,}16 \\ 0{,}17 & 0{,}77 & 0{,}66 \end{pmatrix}$$

e)

$$\begin{pmatrix} 0{,}8 & 0{,}2 & 0{,}1 \\ 0{,}1 & 0{,}7 & 0{,}1 \\ 0{,}1 & 0{,}1 & 0{,}8 \end{pmatrix} \cdot \begin{pmatrix} \frac{55}{42} & -\frac{11}{28} & -\frac{5}{42} \\ -\frac{1}{6} & 1{,}5 & -\frac{1}{6} \\ -\frac{1}{7} & -\frac{1}{7} & \frac{9}{7} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

7)

$$\left(\begin{array}{ccc|ccc} 0{,}5 & 0 & 0{,}5 & 1 & 0 & 0 \\ 0{,}5 & 0{,}5 & 0 & 0 & 1 & 0 \\ 0 & 0{,}5 & 0{,}5 & 0 & 0 & 1 \end{array}\right) \quad \mathrm{I} - \mathrm{II}$$

$$\left(\begin{array}{ccc|ccc} 0{,}5 & 0 & 0{,}5 & 1 & 0 & 0 \\ 0 & -0{,}5 & 0{,}5 & 1 & -1 & 0 \\ 0 & 0{,}5 & 0{,}5 & 0 & 0 & 1 \end{array}\right) \quad \mathrm{II} + \mathrm{III}$$

$$\left(\begin{array}{ccc|ccc} 0{,}5 & 0 & 0{,}5 & 1 & 0 & 0 \\ 0 & -0{,}5 & 0{,}5 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right) \quad \begin{array}{l} 2 \cdot \mathrm{I} - \mathrm{III} \\ 2 \cdot \mathrm{II} - \mathrm{III} \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & -1 & 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right) \cdot (-1)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right)$$

$$\Rightarrow M^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$