LÖSUNGEN (Teil mit Hilfsmitteln)

1) a) $\vec{AB} = \begin{pmatrix} 7-1 \\ 2-0 \\ 6-2 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix}$ $|\vec{AB}| = \sqrt{36+4+16} = \sqrt{56}$ LE

$\vec{DC} = \begin{pmatrix} 5-2 \\ 2-1 \\ 5-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ $|\vec{DC}| = \sqrt{9+1+4} = \sqrt{14}$ LE

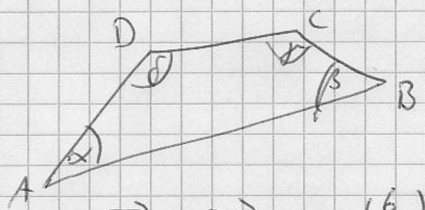
$\vec{AD} = \begin{pmatrix} 2-1 \\ 1-0 \\ 3-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $|\vec{AD}| = \sqrt{3}$ LE

$\vec{BC} = \begin{pmatrix} 5-7 \\ 2-2 \\ 5-6 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}$ $|\vec{BC}| = \sqrt{4+1} = \sqrt{5}$ LE

Die Seiten $\overline{AB}$ und $\overline{DC}$ sind parallel zueinander, da $\vec{AB} = 2 \cdot \vec{DC}$

$\Rightarrow$ ABCD ist ein Trapez

b)



$$\cos\alpha = \frac{\vec{AB} \circ \vec{AD}}{|\vec{AB}| \cdot |\vec{AD}|} = \frac{\begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix} \circ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}{\sqrt{56} \cdot \sqrt{3}} = \frac{6+2+4}{\sqrt{56 \cdot 3}}$$

$$= \frac{12}{\sqrt{168}}$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{12}{\sqrt{168}}\right) \approx 22{,}21°$$

$$\cos\beta = \frac{\vec{BA} \circ \vec{BC}}{|\vec{BA}| \cdot |\vec{BC}|} = \frac{\begin{pmatrix} -6 \\ -2 \\ -4 \end{pmatrix} \circ \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix}}{\sqrt{56} \cdot \sqrt{5}} = \frac{12+4}{\sqrt{56} \cdot \sqrt{5}}$$

$$= \frac{16}{\sqrt{280}}$$

$$\Rightarrow \beta = \cos^{-1}\left(\frac{16}{\sqrt{280}}\right) \approx 17{,}02°$$

$$\cos\gamma = \frac{\vec{CB} \circ \vec{CD}}{|\vec{CB}| \cdot |\vec{CD}|} = \frac{\begin{pmatrix}2\\0\\1\end{pmatrix} \circ \begin{pmatrix}-3\\-1\\-2\end{pmatrix}}{\sqrt{5} \cdot \sqrt{14}} = \frac{-6-2}{\sqrt{5 \cdot 14}}$$

$$= \frac{-8}{\sqrt{70}}$$

$$\Rightarrow \gamma = \cos^{-1}\left(\frac{-8}{\sqrt{70}}\right) \approx 162{,}98°$$

$$\cos\delta = \frac{\vec{DC} \circ \vec{DA}}{|\vec{DC}| \cdot |\vec{DA}|} = \frac{\begin{pmatrix}3\\1\\2\end{pmatrix} \circ \begin{pmatrix}-1\\-1\\-1\end{pmatrix}}{\sqrt{14} \cdot \sqrt{3}} = \frac{-3-1-2}{\sqrt{3 \cdot 14}} = \frac{-6}{\sqrt{42}}$$

$$\Rightarrow \delta = \cos^{-1}\left(\frac{-6}{\sqrt{42}}\right) \approx 157{,}79°$$

c) $\vec{OD} = \vec{OC} + \vec{BA}$

$$= \begin{pmatrix}5\\2\\5\end{pmatrix} + \begin{pmatrix}-6\\-2\\-4\end{pmatrix}$$

$$= \begin{pmatrix}-1\\0\\1\end{pmatrix}$$

$\Rightarrow D(-1/0/1)$

d) $\vec{DA} \circ \vec{DC} = 0$

$$\begin{pmatrix}1-2\\0-1\\2-z\end{pmatrix} \circ \begin{pmatrix}5-2\\2-1\\5-z\end{pmatrix} = 0$$

$$\begin{pmatrix}-1\\-1\\2-z\end{pmatrix} \circ \begin{pmatrix}3\\1\\5-z\end{pmatrix} = 0$$

$$-3-1+(2-z)\cdot(5-z) = 0$$

$$-4+10-2z-5z+z^2 = 0$$

$$6-7z+z^2 = 0$$

$$z^2-7z+6 = 0$$

$$z = 3{,}5 \pm \sqrt{12{,}25-6}$$

$$z = 3{,}5 \pm \sqrt{6{,}25}$$

$$z = 3{,}5 \pm 2{,}5$$

$$z_1 = 1$$
$$z_2 = 6$$

e) $|\overrightarrow{AE}| = |\overrightarrow{BE}|$

$$\left|\begin{pmatrix} 6-1 \\ 1-0 \\ z-2 \end{pmatrix}\right| = \left|\begin{pmatrix} 6-7 \\ 1-2 \\ z-6 \end{pmatrix}\right|$$

$$\left|\begin{pmatrix} 5 \\ 1 \\ z-2 \end{pmatrix}\right| = \left|\begin{pmatrix} -1 \\ -1 \\ z-6 \end{pmatrix}\right|$$

$$\sqrt{25+1+(z-2)^2} = \sqrt{1+1+(z-6)^2} \quad |()^2$$
$$26+(z-2)^2 = 2+(z-6)^2$$
$$26+z^2-4z+4 = 2+z^2-12z+36$$
$$z^2-4z+30 = z^2-12z+38 \quad |-z^2 \;|+12z \;|-30$$
$$8z = 8 \quad |:8$$
$$z = 1$$

f) $|\overrightarrow{AF}| = |\overrightarrow{BF}|$

$$\left|\begin{pmatrix} 1-1 \\ 18-2a-0 \\ a-2 \end{pmatrix}\right| = \left|\begin{pmatrix} 1-7 \\ 18-2a-2 \\ a-6 \end{pmatrix}\right|$$

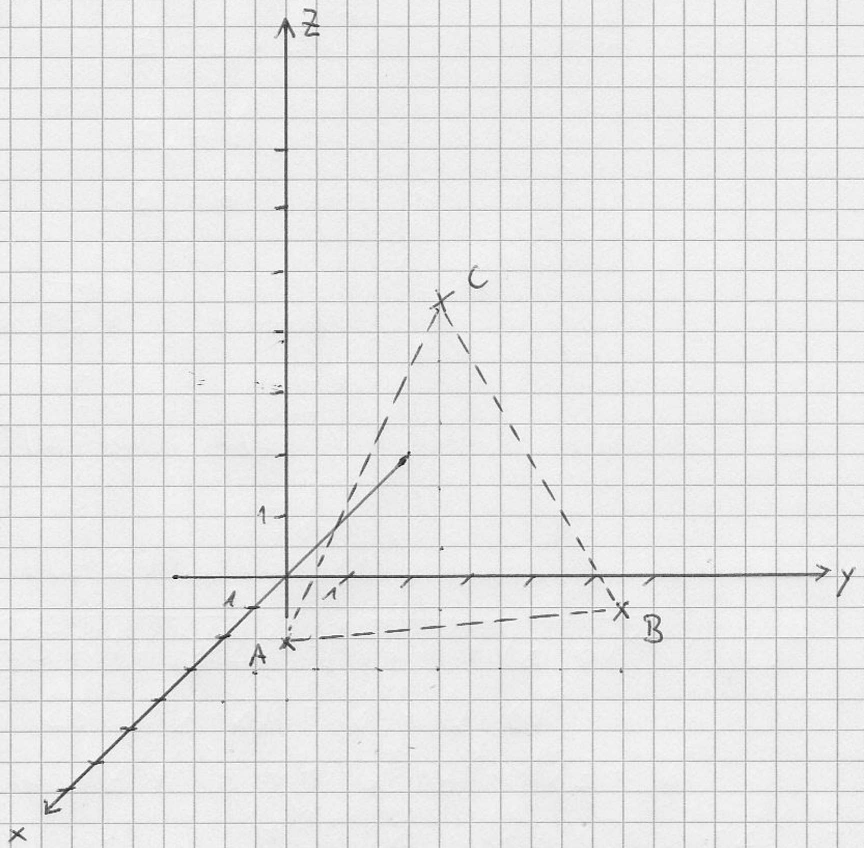
$$\left|\begin{pmatrix} 0 \\ 18-2a \\ a-2 \end{pmatrix}\right| = \left|\begin{pmatrix} -6 \\ 16-2a \\ a-6 \end{pmatrix}\right|$$

$$\sqrt{(18-2a)^2+(a-2)^2} = \sqrt{36+(16-2a)^2+(a-6)^2} \quad |()^2$$

$$(18-2a)^2+(a-2)^2 = 36+(16-2a)^2+(a-6)^2$$
$$324-72a+4a^2+a^2-4a+4 = 36+256-64a+4a^2+a^2-12a+36$$
$$5a^2-76a+328 = 5a^2-76a+328$$

q.e.d.

2) a)



b)

$$\overrightarrow{AB} = \begin{pmatrix} 3-4 \\ 7-2 \\ 1-1 \end{pmatrix} = \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} \quad |\overrightarrow{AB}| = \sqrt{1+25} = \sqrt{26}\ LE$$

$$\overrightarrow{BC} = \begin{pmatrix} 3-3 \\ 4-7 \\ 6-1 \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 5 \end{pmatrix} \quad |\overrightarrow{BC}| = \sqrt{9+25} = \sqrt{34}\ LE$$

$$\overrightarrow{AC} = \begin{pmatrix} 3-4 \\ 4-2 \\ 6-1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \quad |\overrightarrow{AC}| = \sqrt{1+4+25} = \sqrt{30}\ LE$$

c)

$$\cos\alpha = \frac{\overrightarrow{AB} \circ \overrightarrow{AC}}{|\overrightarrow{AB}| \cdot |\overrightarrow{AC}|} = \frac{\begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} \circ \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}}{\sqrt{26} \cdot \sqrt{30}} = \frac{1+10}{\sqrt{26 \cdot 30}}$$

$$= \frac{11}{\sqrt{780}}$$

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{11}{\sqrt{780}}\right) \approx 66{,}80^\circ$$

$$\cos\beta = \frac{\overrightarrow{BA} \circ \overrightarrow{BC}}{|\overrightarrow{BA}| \cdot |\overrightarrow{BC}|} = \frac{\begin{pmatrix}1\\-5\\0\end{pmatrix} \circ \begin{pmatrix}0\\-3\\5\end{pmatrix}}{\sqrt{26} \cdot \sqrt{34}} = \frac{15}{\sqrt{884}}$$

$$\Rightarrow \beta = \cos^{-1}\left(\frac{15}{\sqrt{884}}\right) \approx 59{,}70°$$

$$\gamma = 180° - \alpha - \beta = 180° - 66{,}8° - 59{,}7° = 53{,}5°$$

d) Es handelt sich um kein besonderes Dreieck.

e)

$$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA}$$
$$= \begin{pmatrix}3\\4\\6\end{pmatrix} + \begin{pmatrix}1\\-5\\0\end{pmatrix}$$
$$= \begin{pmatrix}4\\-1\\6\end{pmatrix}$$
$$\Rightarrow D\,(4/-1/6)$$

f)

$$\overrightarrow{OM_{AB}} = 0{,}5 \cdot \left(\overrightarrow{OA} + \overrightarrow{OB}\right)$$
$$= 0{,}5 \cdot \left(\begin{pmatrix}4\\2\\1\end{pmatrix} + \begin{pmatrix}3\\7\\1\end{pmatrix}\right)$$
$$= 0{,}5 \cdot \begin{pmatrix}7\\9\\2\end{pmatrix}$$
$$= \begin{pmatrix}3{,}5\\4{,}5\\1\end{pmatrix}$$
$$\Rightarrow M_{AB}\,(3{,}5/4{,}5/1)$$

$$\overrightarrow{OS} = \overrightarrow{OC} + \frac{2}{3} \cdot \overrightarrow{CM_{AB}}$$
$$= \begin{pmatrix}3\\4\\6\end{pmatrix} + \frac{2}{3} \cdot \begin{pmatrix}3{,}5-3\\4{,}5-4\\1-6\end{pmatrix}$$
$$= \begin{pmatrix}3\\4\\6\end{pmatrix} + \frac{2}{3} \cdot \begin{pmatrix}0{,}5\\0{,}5\\-5\end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix} + \begin{pmatrix} 1/6 \\ 1/6 \\ -10/6 \end{pmatrix}$$

$$= \begin{pmatrix} 19/6 \\ 25/6 \\ 13/6 \end{pmatrix}$$

$$\Rightarrow S\left(19/6 \,/\, 25/6 \,/\, 13/6\right)$$

g) $\overrightarrow{LC} = \begin{pmatrix} 3 - 93/26 \\ 4 - 107/26 \\ 6 - 1 \end{pmatrix} = \begin{pmatrix} -15/26 \\ -3/26 \\ 5 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix}$

$\overrightarrow{LC} \circ \overrightarrow{AB} = \begin{pmatrix} -15/26 \\ -3/26 \\ 5 \end{pmatrix} \circ \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix} = \frac{15}{26} - \frac{3}{26} \cdot 5 = 0$

$\Rightarrow \overrightarrow{LC} \perp \overrightarrow{AB}$

h) Der Vektor $\overrightarrow{LC}$ gibt offenbar die Höhe des Dreiecks wieder, da er orthogonal zu $\overline{AB}$ ist und von C zu $\overline{AB}$ verläuft.

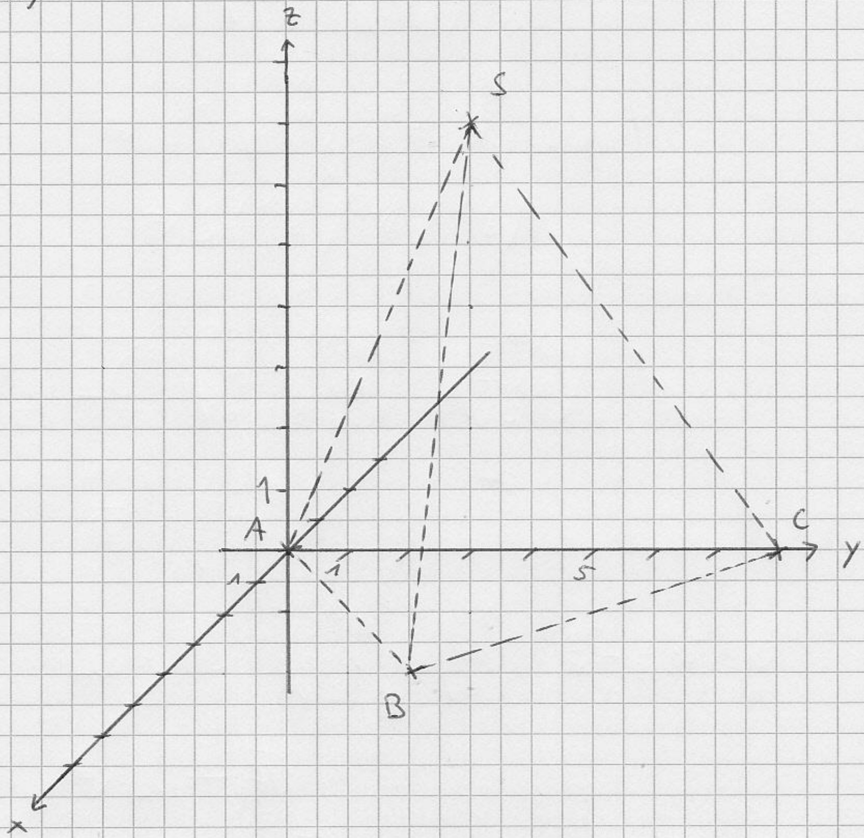
$$A = \frac{1}{2} \cdot |\overrightarrow{AB}| \cdot |\overrightarrow{LC}|$$

$$= \frac{1}{2} \cdot \sqrt{26} \cdot \sqrt{\left(\frac{15}{26}\right)^2 + \left(\frac{3}{26}\right)^2 + 5^2}$$

$$= \frac{1}{2} \cdot \sqrt{26} \cdot \sqrt{\frac{659}{26}}$$

$$\approx 12{,}84 \text{ FE}$$

3) a)



b) $\overrightarrow{AB} = \begin{pmatrix} 4-0 \\ 4-0 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}$ $\quad |\overrightarrow{AB}| = \sqrt{16+16} = \sqrt{32}$ LE

$\overrightarrow{AC} = \begin{pmatrix} 0-0 \\ 8-0 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 0 \\ 8 \\ 0 \end{pmatrix}$ $\quad |\overrightarrow{AC}| = 8$ LE

$\overrightarrow{BC} = \begin{pmatrix} 0-4 \\ 8-4 \\ 0-0 \end{pmatrix} = \begin{pmatrix} -4 \\ 4 \\ 0 \end{pmatrix}$ $\quad |\overrightarrow{BC}| = \sqrt{16+16} = \sqrt{32}$ LE

$\overrightarrow{BA} \circ \overrightarrow{BC} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} \circ \begin{pmatrix} -4 \\ 4 \\ 0 \end{pmatrix} = 16 - 16 = 0$

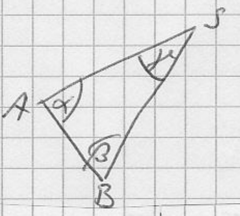
$\Rightarrow \overrightarrow{BA} \perp \overrightarrow{BC}$

Es handelt sich um ein rechtwinkliges und gleichschenkliges Dreieck.

c) $\vec{AB} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix}$ $\qquad |\vec{AB}| = \sqrt{32}$ LE

$\vec{AS} = \begin{pmatrix} 2-0 \\ 4-0 \\ 8-0 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$ $\qquad |\vec{AS}| = \sqrt{4+16+64} = \sqrt{84}$ LE

$\vec{BS} = \begin{pmatrix} 2-4 \\ 4-4 \\ 8-0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 8 \end{pmatrix}$ $\qquad |\vec{BS}| = \sqrt{4+64} = \sqrt{68}$ LE



zu α: $\vec{AB} \circ \vec{AS} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} = 8+16 \neq 0$

zu β: $\vec{BA} \circ \vec{BS} = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix} \circ \begin{pmatrix} -2 \\ 0 \\ 8 \end{pmatrix} = 8 \neq 0$

zu γ: $\vec{SA} \circ \vec{SB} = \begin{pmatrix} -2 \\ -4 \\ -8 \end{pmatrix} \circ \begin{pmatrix} 2 \\ 0 \\ 8 \end{pmatrix} = -4-64 \neq 0$

Es handelt sich um kein besonderes Dreieck.

d) $\cos\varphi = \dfrac{\vec{AB} \circ \vec{AS}}{|\vec{AB}| \cdot |\vec{AS}|} = \dfrac{\begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} \circ \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}}{\sqrt{32} \cdot \sqrt{84}} = \dfrac{8+16}{\sqrt{32 \cdot 84}} = \dfrac{24}{\sqrt{2688}}$

$\Rightarrow \varphi = \cos^{-1}\left(\dfrac{24}{\sqrt{2688}}\right) \approx 62{,}42°$

e) Die Fläche ABC liegt genau in der XY-Ebene. Der Abstand der Spitze vom Boden entspricht daher ihrem z-Wert.

Abstand = 8 LE

f) $A_{\text{Dreieck ABC}} = \frac{1}{2} \cdot |\vec{AB}| \cdot |\vec{BC}| = \frac{1}{2} \cdot \sqrt{32} \cdot \sqrt{32}$
$= 16$ FE

$V_{Pyr} = \frac{1}{3} \cdot \text{Grundfläche} \cdot \text{Höhe}$

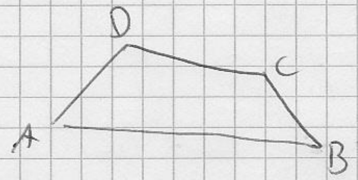
$= \frac{1}{3} \cdot 16 \cdot 8$

$= 42,\bar{6}$ VE

g) $S'(2/4/-8)$

Da S an der xy-Ebene gespiegelt wird, kann sich nur der z-Wert verändern.

4)



$\vec{AB} = \begin{pmatrix} 9-5 \\ -3-0 \\ 1-(-1) \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}$ $|\vec{AB}| = \sqrt{16+9+4} = \sqrt{29}$ LE

$\vec{DC} = \begin{pmatrix} 11-9 \\ 3-6 \\ 4-0 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$ $|\vec{DC}| = \sqrt{4+9+16} = \sqrt{29}$ LE

$\vec{AD} = \begin{pmatrix} 9-5 \\ 6-0 \\ 0-(-1) \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 1 \end{pmatrix}$ $|\vec{AD}| = \sqrt{16+36+1} = \sqrt{53}$ LE

$\vec{BC} = \begin{pmatrix} 11-9 \\ 3-(-3) \\ 4-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ $|\vec{BC}| = \sqrt{4+9} = \sqrt{13}$ LE

Es handelt sich um kein besonderes Viereck.

5)

$\left|\begin{pmatrix} 2 \\ 1 \\ z \end{pmatrix}\right| = 5$

$\sqrt{4+1+z^2} = 5 \quad |()^2$

$5 + z^2 = 25 \quad |-5$

$z^2 = 20 \quad |\sqrt{\ }$

$z_1 = \sqrt{20}$

$z_2 = -\sqrt{20}$

6) $\begin{pmatrix}3\\1\\4\end{pmatrix} \circ \begin{pmatrix}x\\y\\1\end{pmatrix} = 3x + y + 4 = 0$

$3x + y = -4$

$\begin{pmatrix}1\\4\\5\end{pmatrix} \circ \begin{pmatrix}x\\y\\1\end{pmatrix} = x + 4y + 5 = 0$

$x + 4y = -5$

I. $3x + y = -4$

II. $x + 4y = -5$

$$\left(\begin{array}{cc|c}3 & 1 & -4\\1 & 4 & -5\end{array}\right) 3 \cdot \underline{II} - \underline{I}$$

$$\left(\begin{array}{cc|c}3 & 1 & -4\\0 & 11 & -11\end{array}\right)$$

$\Rightarrow 11y = -11$

$y = -1$

$\Rightarrow 3x - 1 = -4$

$3x = -3$

$x = -1$

7) Gesucht: $\vec{c} = \begin{pmatrix}x\\y\\z\end{pmatrix}$

$\vec{c} \circ \vec{a} = \begin{pmatrix}x\\y\\z\end{pmatrix} \circ \begin{pmatrix}2\\3\\-2\end{pmatrix} = 2x + 3y - 2z = 0$

$\vec{c} \circ \vec{b} = \begin{pmatrix}x\\y\\z\end{pmatrix} \circ \begin{pmatrix}2\\1\\0\end{pmatrix} = 2x + y = 0$

I. $2x + 3y - 2z = 0$

II. $2x + y = 0$

Wir formen $\underline{II}$ um:

$$2x + y = 0$$
$$y = -2x$$

Wir setzen das in $\underline{I}$ ein:

$$2x + 3 \cdot (-2x) - 2z = 0$$
$$2x - 6x - 2z = 0$$
$$-4x - 2z = 0$$
$$-4x = 2z$$
$$-2x = z$$

$\Rightarrow$ Für $\vec{c}$ gilt $\vec{c} = \begin{pmatrix} x \\ -2x \\ -2x \end{pmatrix} = x \cdot \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

Wir testen $\vec{c} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$:

$$\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \circ \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} = 2 - 6 + 4 = 0 \quad \checkmark$$

$$\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \circ \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = 2 - 2 = 0 \quad \checkmark$$

Ergebnis: $\vec{c} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

8) $r \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + s \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + t \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right) \; \underline{I} - \underline{II}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right) \; \underline{II} + \underline{III}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{array}\right)$$

$\Rightarrow 2t = 1$ $\quad \Rightarrow -s + 0{,}5 = 0$ $\quad \Rightarrow r + 0{,}5 = 1$

$t = 0{,}5$ $\quad s = 0{,}5$ $\quad r = 0{,}5$

9) a) $\overrightarrow{OM_{AB}} = 0{,}5 \cdot (\overrightarrow{OA} + \overrightarrow{OB})$

$$= 0{,}5 \cdot \left(\begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \\ 6 \end{pmatrix} \right)$$

$$= 0{,}5 \cdot \begin{pmatrix} 5 \\ 8 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} 2{,}5 \\ 4 \\ 5{,}5 \end{pmatrix}$$

$\Rightarrow M_{AB} \, (2{,}5 / 4 / 5{,}5)$

$\overrightarrow{OS} = \overrightarrow{OC} + \frac{2}{3} \cdot \overrightarrow{CM_{AB}}$

$$= \begin{pmatrix} 4 \\ 4 \\ 10 \end{pmatrix} + \frac{2}{3} \cdot \begin{pmatrix} 2{,}5 - 4 \\ 4 - 4 \\ 5{,}5 - 10 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 4 \\ 10 \end{pmatrix} + \frac{2}{3} \cdot \begin{pmatrix} -1{,}5 \\ 0 \\ -4{,}5 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 4 \\ 7 \end{pmatrix}$$

$\Rightarrow S \, (3/4/7)$

b) $\overrightarrow{AB} = \begin{pmatrix} 3 - 2 \\ 5 - 3 \\ 6 - 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \quad |\overrightarrow{AB}| = \sqrt{1+4+1} = \sqrt{6} \text{ LE}$

$\overrightarrow{AC} = \begin{pmatrix} 4 - 2 \\ 4 - 3 \\ 10 - 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \quad |\overrightarrow{AC}| = \sqrt{4+1+25} = \sqrt{30} \text{ LE}$

$\overrightarrow{BC} = \begin{pmatrix} 4 - 3 \\ 4 - 5 \\ 10 - 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \quad |\overrightarrow{BC}| = \sqrt{1+1+16} = \sqrt{18} \text{ LE}$

c)

zu α: $\cos \alpha = \dfrac{\overrightarrow{AB} \circ \overrightarrow{AC}}{|\overrightarrow{AB}| \cdot |\overrightarrow{AC}|} = \dfrac{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \circ \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}}{\sqrt{6} \cdot \sqrt{30}} = \dfrac{2+2+5}{\sqrt{6 \cdot 30}}$

$$= \frac{9}{\sqrt{180}}$$

$\Rightarrow \alpha = \cos^{-1}\left(\frac{9}{\sqrt{180}}\right) \approx 47{,}87°$

zu β: $\cos\beta = \frac{\vec{BA} \circ \vec{BC}}{|\vec{BA}| \cdot |\vec{BC}|} = \frac{\begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix} \circ \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}}{\sqrt{6} \cdot \sqrt{18}} = \frac{-1+2-4}{\sqrt{6 \cdot 18}}$

$= \frac{-3}{\sqrt{108}}$

$\Rightarrow \cos\beta = \frac{-3}{\sqrt{108}}$

$\beta = \cos^{-1}\left(\frac{-3}{\sqrt{108}}\right) \approx 106{,}78°$

zu γ: $\gamma = 180° - \alpha - \beta = 180° - 47{,}87° - 106{,}78° = 23{,}35°$

d) $\vec{OD} = \vec{OC} + \vec{BA} = \begin{pmatrix} 4 \\ 4 \\ 10 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 9 \end{pmatrix}$

$\Rightarrow D(3/2/9)$

e) Eine Möglichkeit: $C(4/4/z)$

$|\vec{AC}| = |\vec{BC}|$

$\left|\begin{pmatrix} 4-2 \\ 4-3 \\ z-5 \end{pmatrix}\right| = \left|\begin{pmatrix} 4-3 \\ 4-5 \\ z-6 \end{pmatrix}\right|$

$\left|\begin{pmatrix} 2 \\ 1 \\ z-5 \end{pmatrix}\right| = \left|\begin{pmatrix} 1 \\ -1 \\ z-6 \end{pmatrix}\right|$

$\sqrt{4+1+(z-5)^2} = \sqrt{1+1+(z-6)^2} \quad |()^2$

$5 + (z-5)^2 = 2 + (z-6)^2$

$5 + z^2 - 10z + 25 = 2 + z^2 - 12z + 36$

$z^2 - 10z + 30 = z^2 - 12z + 38$

$-10z + 30 = -12z + 38$

$2z + 30 = 38$

$2z = 8$

$z = 4$

$\Rightarrow C(4/4/4)$