LÖSUNGEN (hilfsmittelfreier Teil)

1a)
$$6^2 + 6a + 4 = 0$$
$$36 + 6a + 4 = 0$$
$$40 + 6a = 0$$
$$6a = -40$$
$$a = -\frac{40}{6} = -\frac{20}{3}$$

b) N.B.: $f_a'(x) = 0$
$$2x + a = 0$$
$$2x = -a$$
$$x = -\frac{a}{2}$$

H.B.: $f_a'(x) = 0 \wedge f_a''(x) \neq 0$
$$f_a''\left(-\frac{a}{2}\right) = 2 > 0$$
$\Rightarrow$ Min. bei $x = -\frac{a}{2}$

y-Wert:
$$f_a\left(-\frac{a}{2}\right) = \left(-\frac{a}{2}\right)^2 + a\cdot\left(-\frac{a}{2}\right) + 4$$
$$= \frac{a^2}{4} - \frac{a^2}{2} + 4$$
$$= -\frac{a^2}{4} + 4$$
$\Rightarrow$ ES $\left(-\frac{a}{2} \middle/ -\frac{a^2}{4} + 4\right)$

c)
$$x = -\frac{a}{2}$$
$$-2x = a$$
$$\Rightarrow y = -\frac{a^2}{4} + 4 = -\frac{(-2x)^2}{4} + 4 = -\frac{4x^2}{4} + 4 = -x^2 + 4$$
$$\Rightarrow O(x) = -x^2 + 4$$

d) $\int_0^1 f_a(x)\,dx = \left[\frac{1}{3}x^3 + \frac{1}{2}ax^2 + 4x\right]_0^1$

$= \frac{1}{3} + \frac{1}{2}a + 4 - 0$

$= \frac{1}{3} + \frac{1}{2}a + 4$

$= \frac{13}{3} + \frac{1}{2}a$

e) $\frac{13}{3} + \frac{1}{2}a = 10 \quad \Big| - \frac{13}{3}$

$\frac{1}{2}a = 10 - \frac{13}{3}$

$\frac{1}{2}a = \frac{30}{3} - \frac{13}{3}$

$\frac{1}{2}a = \frac{17}{3} \quad \Big| \cdot 2$

$a = \frac{34}{3}$

f) $f_a(x) = x^2 + ax + 4$

$f_a'(x) = 2x + a$

$f_a'(0) = 0 + a = a$

$\Rightarrow \quad t(x) = a \cdot x + b$

$f_a(0) = 4 \Rightarrow t(0) = 4$

$a \cdot 0 + b = 4$

$b = 4$

$\Rightarrow t(x) = a \cdot x + 4$

g) $x^2 + a_1 x + 4 = x^2 + a_2 x + 4 \quad | -x^2 | -4$

$a_1 x = a_2 x \quad | -a_2 x$

$a_1 x - a_2 x = 0$

$x \cdot \underbrace{(a_1 - a_2)}_{\neq 0} = 0$

$\Rightarrow x = 0$

$f_a(0) = 4$

$\Rightarrow$ Es gibt einen gemeinsamen Punkt: $P(0/4)$.

2) a) $x^3 - 6x^2 - 7x = 0$

$x \cdot (x^2 - 6x - 7) = 0$

$x_1 = 0 \qquad x^2 - 6x - 7 = 0$

$x = 3 \pm \sqrt{9 + 7}$

$x = 3 \pm \sqrt{16}$

$x = 3 \pm 4$

$x_2 = -1$

$x_3 = 7$

Nullstellen: $x_1 = 0$

$x_2 = -1$

$x_3 = 7$

b) geratene NS: $x_1 = -1$

(denn $(-1)^3 + 9\cdot(-1)^2 + 23\cdot(-1) + 15$
$= -1 + 9 - 23 + 15$
$= 0$)

$$(x^3 + 9x^2 + 23x + 15) : (x+1) = x^2 + 8x + 15$$
$$\begin{array}{l} -(x^3 + x^2) \\ \hline \quad 8x^2 + 23x \\ \quad -(8x^2 + 8x) \\ \hline \qquad 15x + 15 \\ \qquad -(15x + 15) \\ \hline \qquad\qquad 0 \end{array}$$

$x^2 + 8x + 15 = 0$

$x = -4 \pm \sqrt{16-15}$

$x = -4 \pm 1$

$x_2 = -5$

$x_3 = -3$

Nullstellen: $x_1 = -1$; $x_2 = -3$; $x_3 = -5$

c) $1 - \frac{5}{x^2} + \frac{4}{x^4} = 0 \quad | \cdot x^4$

$x^4 - 5x^2 + 4 = 0 \quad | x^2 = z$

$z^2 - 5z + 4 = 0$

$z = 2{,}5 \pm \sqrt{6{,}25 - 4}$

$z = 2{,}5 \pm \sqrt{2{,}25}$

$z = 2{,}5 \pm 1{,}5$

$z_1 = 1 \qquad z_2 = 4 \qquad | z = x^2$

$x^2 = 1 \qquad x^2 = 4 \qquad | \sqrt{\ }$

$x_1 = 1;\ x_2 = -1;\ x_3 = 2;\ x_4 = -2$

3a) $F_1(x) = \frac{1}{6}x^6 + \frac{2}{4}x^4 + 3x$

$F_2(x) = \frac{1}{6}x^6 + \frac{1}{2}x^4 + 3x + 7$

b) $F_1(x) = 9x$

$F_2(x) = 9x + 8$

c) $F_1(x) = \frac{1}{7}x^7 + \frac{7}{3}x^3 + x^2 + 4x$

$F_2(x) = \frac{1}{7}x^7 + \frac{7}{3}x^3 + x^2 + 4x + 1769$

d) $f(x) = \frac{1}{x^2} = x^{-2}$

$F_1(x) = \frac{1}{-1}x^{-1} = -x^{-1} = \frac{-1}{x}$

$F_2(x) = \frac{-1}{x} + 1804$

e) $f(x) = \frac{-3}{x^3} = -3 \cdot x^{-3}$

$F_1(x) = -3 \cdot \frac{1}{-2}x^{-2} = \frac{3}{2}x^{-2} = \frac{3}{2} \cdot \frac{1}{x^2} = \frac{3}{2x^2}$

$F_2(x) = \frac{3}{2x^2} + 1799$

f) $f(x) = \sqrt{x^3} = x^{\frac{3}{2}}$

$F_1(x) = \frac{2}{5}x^{\frac{5}{2}} = \frac{2}{5} \cdot \sqrt{x^5}$

$F_2(x) = \frac{2}{5} \cdot \sqrt{x^5} + 1809$

4) $F(x) = \frac{1}{2}x^2 + 4x + c$

$F(2) = \frac{1}{2} \cdot 2^2 + 4 \cdot 2 + c = 20$

$2 + 8 + c = 20$

$c = 10$

$\Rightarrow F(x) = \frac{1}{2}x^2 + 4x + 10$

5) $A_{Rechteck} = y \cdot x$ (Hauptbed.)

$y = -\frac{1}{3}x^2 + 1$ (Nebenbed.)

$A(x) = \left(-\frac{1}{3}x^2 + 1\right) \cdot x$ (Zielf.)

$= -\frac{1}{3}x^3 + x$

N.B.: $A'(x) = 0$

$-x^2 + 1 = 0$

$-x^2 = -1$

$x = 1$ ($x = -1$ liegt nicht im 1. Quadranten)

H.B.: $A'(x) = 0 \wedge A''(x) \neq 0$

$A''(x) = -2x$

$A''(1) = -2 < 0$

$\Rightarrow$ Max. bei $x = 1$

Ränder: einzige ES ✓

y-Wert: $y = -\frac{1}{3} \cdot 1^2 + 1 = -\frac{1}{3} + 1 = \frac{2}{3}$

$\Rightarrow P\left(1 / \frac{2}{3}\right)$

6) Extremstellen: N.B: $f'(x)=0$

$$f'(x) = 3ax^2 + 2bx$$

$$3ax^2 + 2bx = 0$$

$$x\,(3ax + 2b) = 0$$

$$x_1 = 0 \qquad 3ax = -2b$$

$$x = \frac{-2b}{3a}$$

H.B.: $f'(x) = 0 \wedge f''(x) \neq 0$

$$f''(x) = 6ax + 2b$$

$$f''(0) = 2b \neq 0 \Rightarrow \text{ES bei } x = 0$$

$$f''\left(-\frac{2b}{3a}\right) = 6a \cdot \frac{-2b}{3a} + 2b$$

$$= -4b + 2b$$

$$= -2b$$

$$\neq 0 \Rightarrow \text{ES bei } x = \frac{-2b}{3a}$$

Wendestelle: N.B.: $f''(x) = 0$

$$6ax + 2b = 0$$

$$6ax = -2b$$

$$x = \frac{-2b}{6a} = \frac{-b}{3a}$$

H.B.: $f''(x) = 0 \wedge f'''(x) \neq 0$

$$f'''(x) = 6a$$

$$f'''\left(-\frac{b}{3a}\right) \neq 0 \Rightarrow \text{WS bei } x = -\frac{b}{3a}$$

Ergebnis: $x = \frac{-b}{3a}$ liegt genau in der Mitte zwischen $x = 0$ und $x = \frac{-2b}{3a}$.

7)
$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 2 & -1 & 1 & 1 \\ 2 & 2 & -1 & 5 \end{array}\right) \begin{array}{l} \\ 2\cdot \mathrm{I} - \mathrm{II} \\ 2\cdot \mathrm{I} - \mathrm{III} \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 3 & 3 & 9 \\ 0 & 0 & 5 & 5 \end{array}\right)$$

$\Rightarrow 5z = 5$

$z = 1$

$\Rightarrow 3y + 3 = 9$

$3y = 6$

$y = 2$

$\Rightarrow x + 2 + 2 = 5$

$x = 1$

Ergebnis: $x = 1$; $y = 2$; $z = 1$

8) Nullstellen: $-x^2 + 4x = 0$

$x^2 - 4x = 0$

$x = 2 \pm \sqrt{4-0}$

$x = 2 \pm 2$

$x_1 = 0$

$x_2 = 4$

$$A = \int_0^4 -x^2 + 4x \, dx = \left[-\frac{1}{3}x^3 + 2x^2\right]_0^4$$

$= -\frac{1}{3}\cdot 4^3 + 2\cdot 4^2 - 0$

$= -\frac{64}{3} + 16\cdot 2$

$= -\frac{64}{3} + 32$

$= -\frac{64}{3} + \frac{96}{3}$

$= \frac{32}{3}$

$\Rightarrow A = 32/3$ FE